

The Pragmatic Dimension of Net Theory

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Introduction

In the area of semiotics, or the science of communication by sending and receiving messages, pragmatics is that apparently boundless part of the whole science which lies beyond the well-established and sufficiently formalized small parts of syntax and semantics; that part which lies beyond the ideas of a correctly composed sentence and its standardized "meaning".

In the wider context of organisation and technology, to which we shall confine ourselves, pragmatics comprises, by extension, those structures of immediate concern to persons, which show up as questions relevant to technology.

Characteristic topics in this context are e.g.

- the persons' specific change (not only of knowledge, but also and mainly) of basic attitudes and of their behaviour, by experience (e.g. by receiving a message from a known source)
- exercise of power, influence, control
- conflict, disagreement with others, own doubts
- diversity of interests, purposes, beliefs and preferences
- responsibilities, sanctions, laws and freedoms
- contracts and their reliability

We do not claim this to be an exhaustive list of human concerns. What we do urge is that these concerns must not be treated as diffuse matters of opinion:

In Informatics as well as in the practice of information technology, we see a strong need to direct one's attention beyond the increase of throughput, short-range correctness and decreasing probabilistic failure rates: towards establishing entirely new ideas on reliability of complex systems and on "precision in a world

of imprecision"; and beyond the (pragmatically diffuse) "data" processing towards a technologically feasible *document* processing, strictly following certain very permissive rules, but these rules would have to be known and accepted rules — well understood by *all concerned persons* ("communication disciplines").

Some Pragmatic Aspects of Net Theory

In net theory an attempt has been made to build solid foundations for a rational and practicable theory dealing with the concerns named above.

Net theory has incorporated a touch of Pragmatics from its very beginning: it demands respect for e.g.

- limitation of all resources
- inherent imprecision of measurement
- partial independence of actions and decisions
- existence of illusions ("discrete" and "continuous" models)

as the core of its "pragmatic" attitude.

These concepts clearly conform to a "common sense" view of the world. On second thought, however, they are seen to lead to rather far-reaching implications and consequences for the principles of building mathematical models of real-world affairs.

Limitation of all Resources

Consider for example the first point mentioned above, the "limitation of all resources". Theoretical Informatics traditionally assumes an unbounded supply of resources, such as infinite Turing tapes or the possibility of arbitrarily precise computations with real numbers, whereas resource questions are treated as a problem of actual implementation only.

A theory, however, which incorporates pragmatic concerns from the very beginning has to classify all kinds of resources according to their pragmatic status, availability and relevance.

Given a collection of objects, pragmatically a new order of magnitude is reached by considering factual or conceivable combinations of these objects. In Mathematics this has a correspondence in the axiom of power sets, an axiom whose pre-eminent status is generally recognized.

Thus as "natural" orders of magnitude we can regard

$$G_0 := 0, \quad G(n+1) := 2^{G_n}; \quad \text{so that}$$

G_0	G_1	G_2	G_3	G_4	G_5	G_6
0	1	2	4	16	65536	$\sim 10^{20000}$

G describes "pragmatic" upper bounds for quantities in different spheres of human concern:

G4: elements of a *fully understandable* irregular net

G5: bits for human pattern recognition *in one glance*

G6: System components
(quarks, ..., galaxies, *brain*, computer)
steps in sequence, *in life*
~~coexisting separate objects~~
bits in a *document*

G7: *alternatively implementable documents*

G_8 might thus be considered as an upper bound for the number of conceivable ideas. For G_9 we know of no example of practical relevance.

Some Typical Pragmatic Results

Let us now turn to some typical pragmatic results obtained from net theory.

- A single bit (of varying value) describing a state-variable at place A cannot be represented by a factually equivalent bit at a place $A' \neq A$.

(The factual interconnection between variable data and what they represent does not exist on the bit level.)

Interpreted in a strictly physical sense this is trivial because of the finite propagation speed of signals. Using net-theory it can be shown by applying the calculus of *facts*.

- An agency (a person) producing measurement data must be granted 1 bit of freedom (must be given a free choice between two alternatives of classification of its observation) for each measurement in order to perform reliably.

(*Reliable* measurement data contain 1 bit of *non-objective* information, reflecting the free *choice* of the observer.)

For a detailed treatment of these two issues we have to refer to the literature. With the following examples we shall try to convey to you at least a flavour of the *methods* provided by net theory for applications of pragmatic concern:

1. A bit cannot exert influence without changing its pragmatic status.
2. There are two basically different kinds of controversy
(Resource conflict and contradiction of assertions).
3. Symmetric self-stabilizing contracts do exist.
(No reliance on a superior and non-partisan authority is needed for those.)

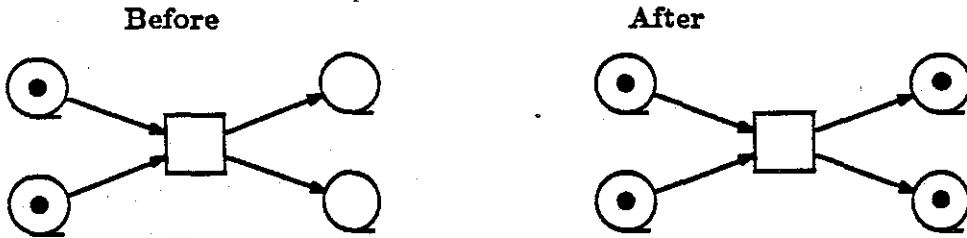
But first it might be useful to pause a moment to recall that the foundations of net theory themselves are in fact closely related to pragmatic concepts.

In the framework of net theory pragmatical aspects are generally considered on a very high level interpretation of nets, illustrated e.g. by the following pairs of concepts:

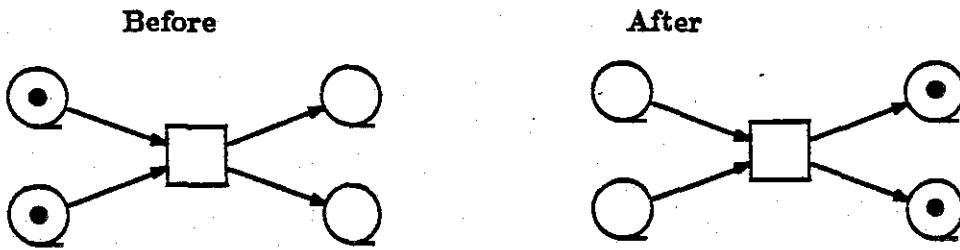
(S)	[T]	
STATES	TRANSITIONS	<i>for introduction</i>
CONDITIONS	EVENTS	<i>for axiomatisation</i>
OPEN <i>s.</i>	CLOSED <i>s.</i>	<i>for mathematical treatment</i>
COMPOUNDS	REACTIONS	<i>in chemistry:</i> <i>e.g. defining "equilibrium" by</i> <i>T-complementation; Analysis of</i> <i>reactions as dual to that of chem.</i> <i>compounds ...</i>
...	...	
...	...	
...	...	
CHANNELS	AGENCIES	<i>for communication</i>
PRAGMATIC STATUS	PRAGMATIC TRANSFORMATION	<i>for specification of pragmatic aspects</i>

However, in effect it turns out that the pragmatic view is the fundamental one; *it is constitutive for net theory:*

Let us look at an early example (1960) concerning a computer action involving several tapes. Technically (on the physical level) the input tapes remain unaffected by the process:

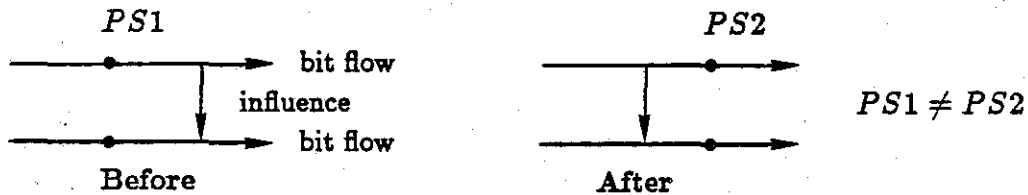


In contrast, in the organizational (pragmatic) view the input tapes change their status from "to be processed" to "processed":

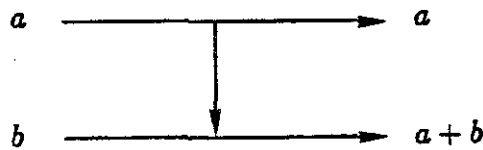


So, in particular it is the pragmatic view which conforms to the extensionality principle for elementary changes.

1. A Bit cannot exert Influence without changing its Pragmatic Status

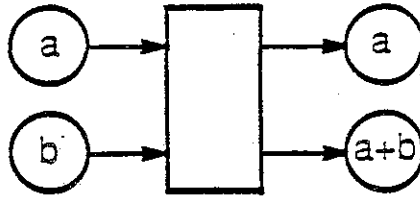


To make the discussion less abstract, assume the above to be

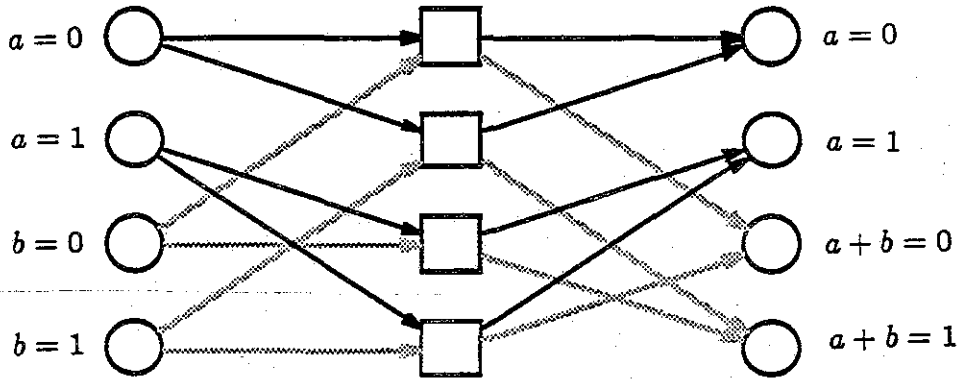


where a and b are Boolean variables and $+$ is the sum modulo 2. Now this is an example of an *information flow graph*. In fact it is the smallest non-trivial one.

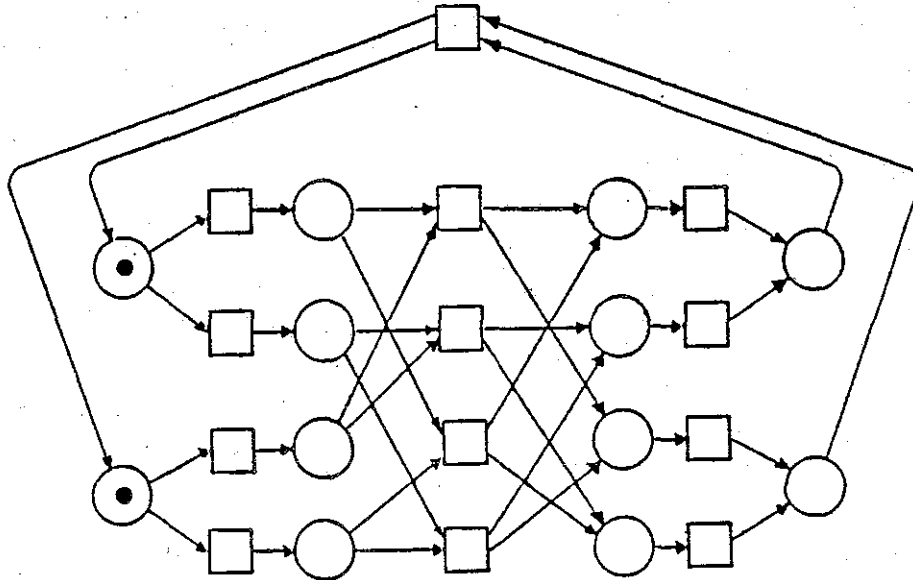
It is often also denoted by



suggesting an unfolding into a condition/event system. The smallest net supporting such an unfolding is

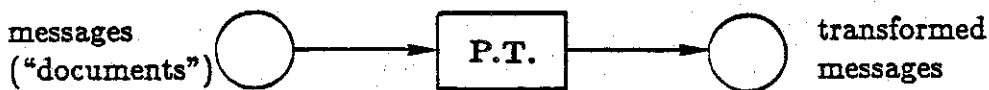
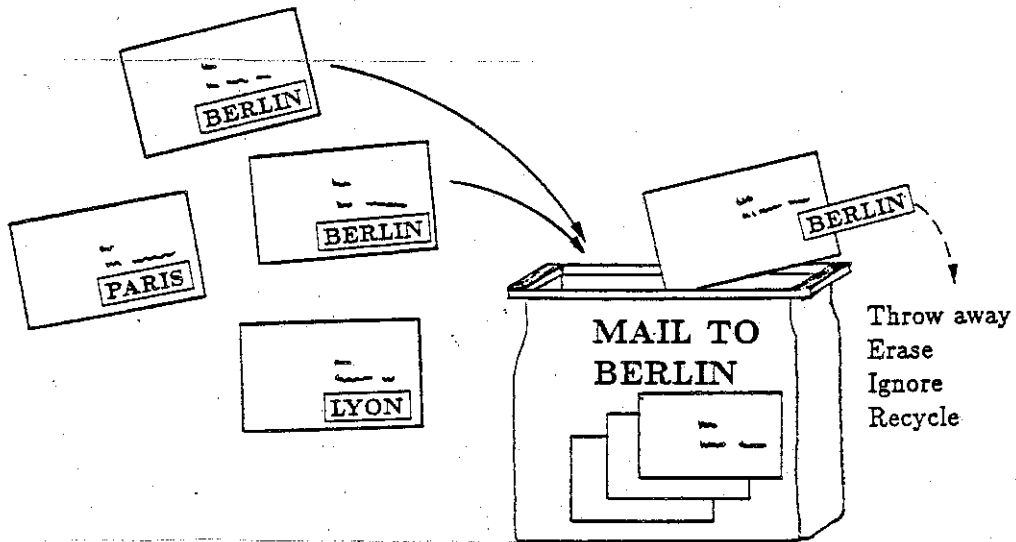


which, embedded into a cyclic condition/event system, gives rise to



Observe that the transformation of input bits into output bits is *reversible* and *impartial*, i.e. it constitutes a *pragmatic equivalence transformation*.

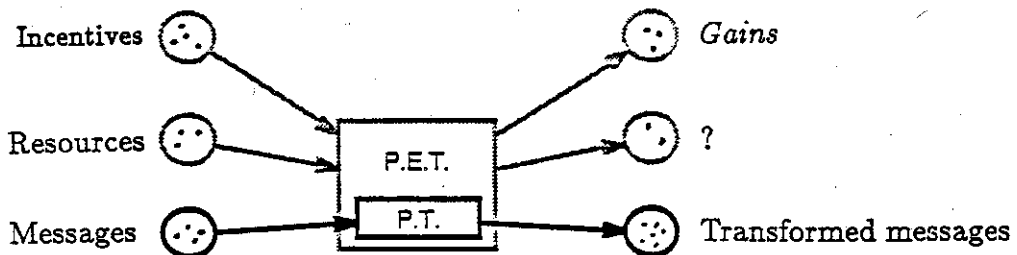
On a higher level the sending of messages ("documents") can be considered as a typical pragmatic transformation (P.T.); e.g.:



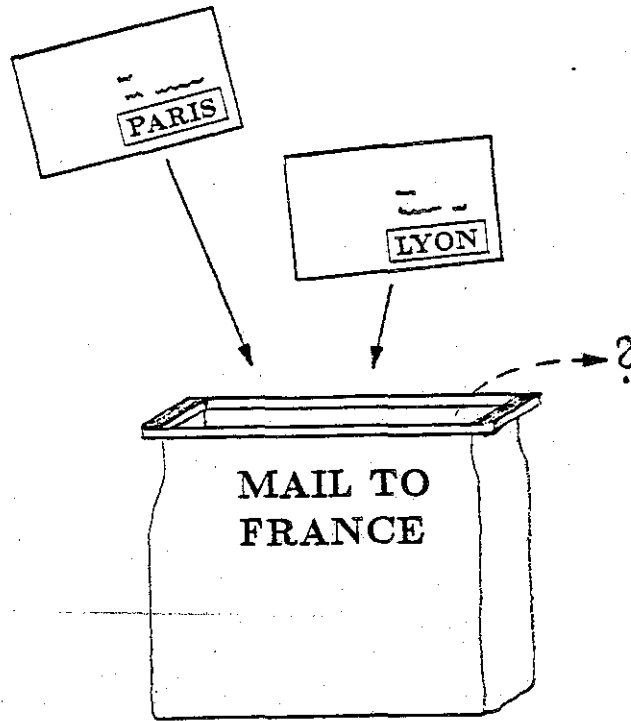
In general this transformation cannot be effectuated without the supply of additional resources and incentives, which are coincidentally transformed into "gains" and other items, as e.g. used-up resources.

Following this idea one is lead to:

Every pragmatic transformation can be embedded into a pragmatic equivalence transformation(P.E.T.).

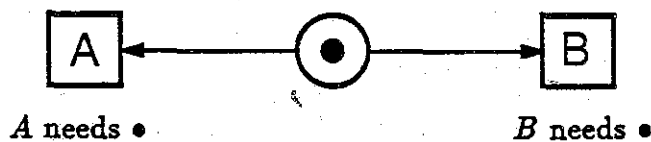


As another example you might consider:



2. Two basically different Kinds of Controversy

One kind of controversy between persons, say *A* and *B*, is the contention for a scarce resource:



Relative to the environment this may be a *situation of conflict*, or of *confusion*.

Quite another kind of controversy is the disagreement of assertions: Assume *A* asserts the proposition *x*, whereas *B* asserts *non-x* (i.e. *B* denies *x*). This can be represented by

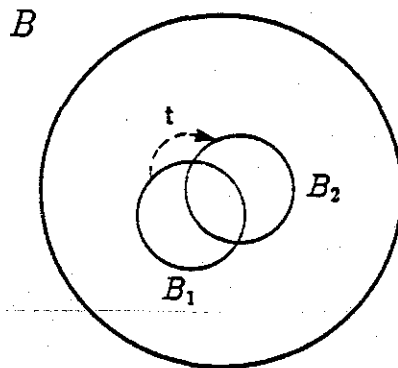


It is not possible to transform a contention for resources into a disagreement of assertions, nor the other way round; they are indeed basically different. Methods for

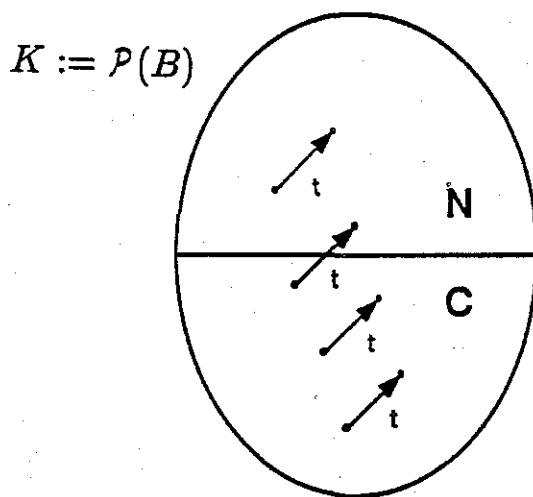
removing controversies (e.g. by negotiation) arise from the theories of *synchronic structure* and *enlogic structure* of systems. They consist in *synchronic* respectively *enlogic compression /decompression*.

To give a taste of these methods we shall look at some aspects of enlogy relevant for reducing the second type of controversy — disagreement about assertions.

For this we need some formal definitions. A *transitional form* t of a condition/event system $\Sigma = (B, E, F, C)$ is a *conceivable* transition characterized by a pair of subsets (B_1, B_2) of B such that $B_1 = {}^*t$ and $B_2 = t^*$.

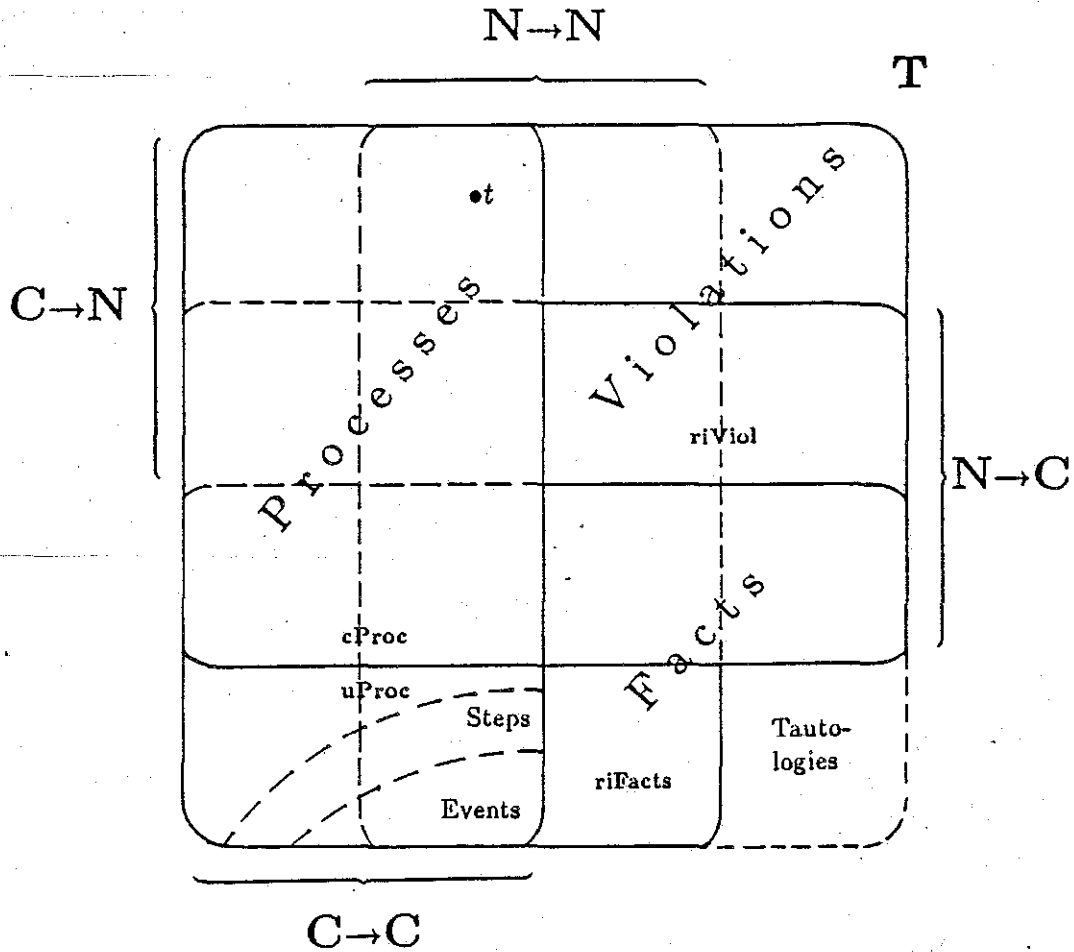


The transitional forms can be classified according to how they would transform the constellations of Σ . By the extensionality principle, a pair (B_1, B_2) such that $B_1 \cap B_2 \neq \emptyset$, does not represent a constellation transformation at all. A *pure* transitional form t (i.e. where ${}^*t \cap t^* = \emptyset$), however, could for example be of the type

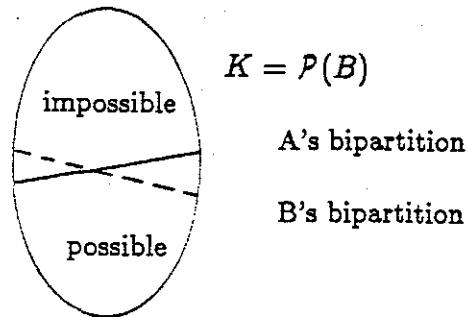


meaning that t transforms at least one case into a case, one case into a non-case, one non-case into a non-case and no non-case into a case.

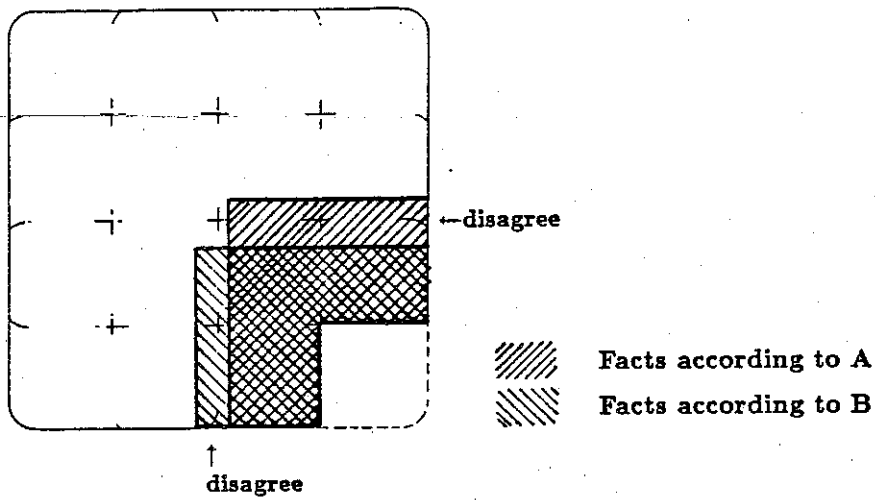
Similarly every transitional form is seen to belong uniquely to one of 16 mutually disjoint classes; thus characterizing the *enlogic structure* of Σ .



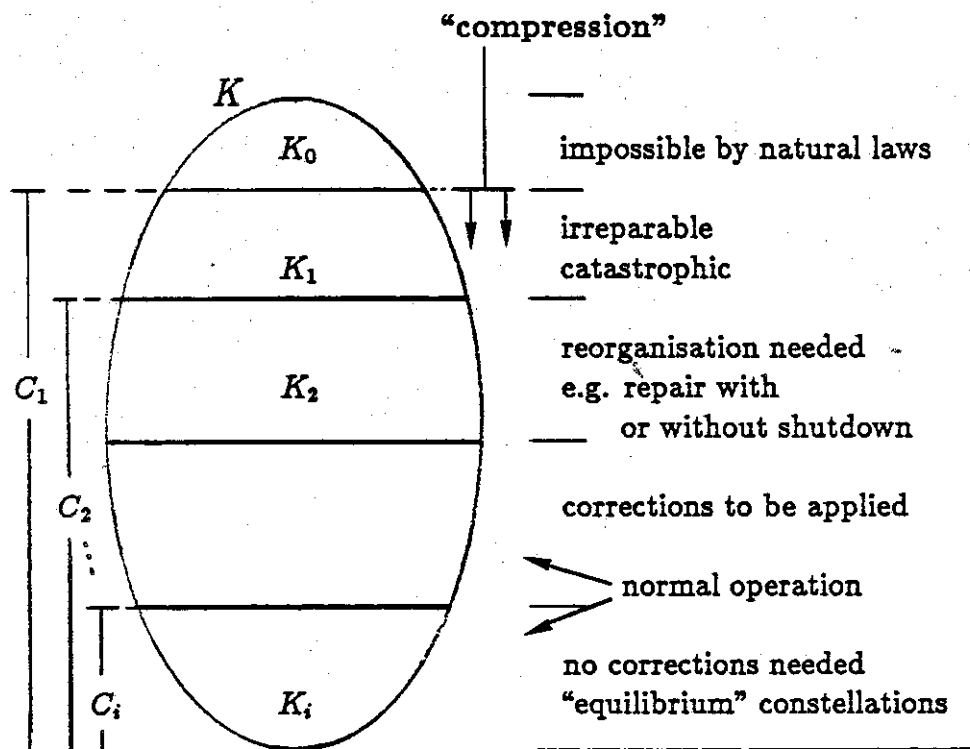
We cannot make a detailed discussion of the enlogic structure here. Rather we shall return to *A*'s and *B*'s disagreement about the assertion x . In terms of enlogy, *A* holds \boxed{A} to be a fact of the surrounding system thus making \boxed{B} a non-fact. And vice versa for *B*. This is tantamount to saying that they disagree upon the partition of $K = \mathcal{P}(B)$ into cases and non-cases:



which in turn leads to a *multiple enlogic structure*



We reduce all kinds of multiple enlogic structures by considering the simple idea of *Enlogic compression*. This concept is illustrated by the following example of a system where different layers of the class of constellations can be distinguished.



$$\Lambda_j : K_j \neq 0$$

A simple everyday example: Suppose A has bought something, say a watch, from B . Now A claims that the watch does not work well: it loses one second a day. Therefore he requires B to have it repaired. B , however, refuses, saying that this is "perfectly normal". In the following dispute A tries to "press down" the borderline between K_i and K_{i-1} in order to show that the constellation clearly belongs to K_{i-1} , whereas B motivated by his interests attempts to push the borderline upwards.

One can easily find examples more typical for the situation described above. But in any case the localisation of the borderlines may be a matter of disagreement, due to diversity of interests and different experience, and should therefore explicitly be subject to negotiation.

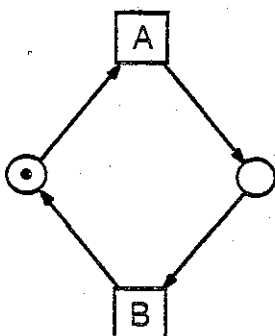
Negotiations about the different borders as in the figure above can be made independent of each other; i.e. it is sufficient to look successively at the different bipartitions $C_j, K - C_j$ instead of immediately at the full partition K_0, \dots, K_i . Thus the number of types of transitional forms to be considered according to the full multiple enlogic structure may be reduced significantly (e.g. for $i = 4$ from 2^{25} to 2^{13}).

3. Symmetric, Self-Stabilizing Contracts without Necessity of Appeal

As a last example here of application of net theory to pragmatic concerns we shall outline a method to obtain contracts in situations where there is no superior authority to appeal to.

Assume A and B e.g. to be representatives of two countries negotiating over some vital issue, such as a practicable way to control disarmament.

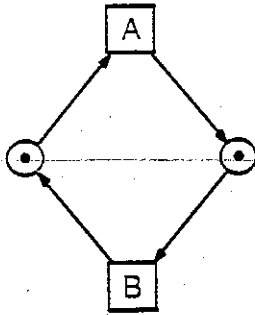
An asymmetric model like



where A and B denote the corresponding acts of contract fulfillment

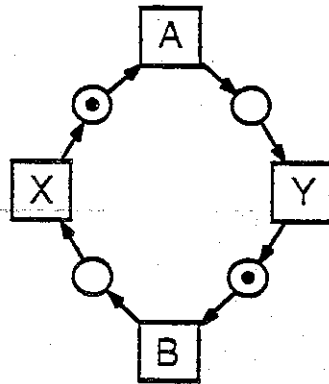
will not be accepted by A since he is expected to take the first step.

Also



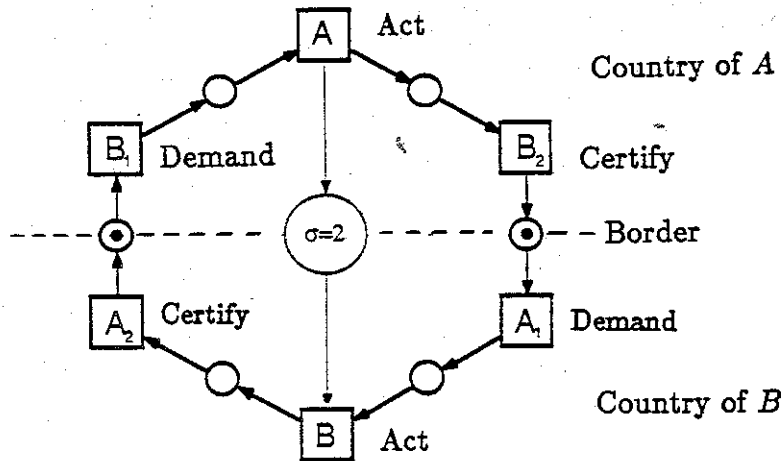
though symmetric, is no solution since *A* and *B* will wait for each other to take the first step; a situation of deadlock!

At first sight, it might seem that both problems are avoided in



However, *A* and *B* will suspect *X* and *Y* of being partial to *B* respectively *A*, so nothing is really gained.

Hence a further enlargement of the model is required:



We claim that this model does represent a solution:

Assume that A_1 , a representative of *A*, hands over a demand for a fulfillment step to *B*. When *B* has fulfilled the demand this is certified by A_2 , another

representative of A . Together with this acknowledgement a representative of B , (i.e. B_1), can now demand a fulfillment step of A etc. —On the other hand B_1 could equally well have started the sequence of actions: the situation is symmetric. In fact no order at all is prescribed; A_1 and B_1 can present their respective demands concurrently, so that also the respective acts of fulfillment do not depend on one another. The independence is nonetheless bounded: In any unbiased report on some present state of affairs A will always be at most one step ahead of B and vice versa. This is expressed by the *synchronic distance* $\sigma = 2$ between A and B .

Remark: Technically $\sigma = 2$ follows from the fact that the net is marked with 2 tokens. If the net were marked with, say 5 tokens, we would have $\sigma = 1$.

For a detailed treatment of the notions synchronic distance and of synchronic structure in general we again have to refer to the literature.

As a final remark let us mention that the synchronic structure as well as the enlogic structure of a system is conceptually one order of magnitude G larger (in the sense discussed before) than the system net.

Conclusion

There can hardly be any doubt about the usefulness of formalizing pragmatic affairs, at least to the extent to which they can influence the actual construction of hardware and software, but also manuals and organizational conventions. As we have hinted, this extent may be considerable. So the crucial question becomes: Is it at all *feasible* to formalize pragmatic affairs? In our opinion, the tools and methods provided by general net theory give the possibility to do so with conceptual depth and mathematical rigour. Such rigour must of course stop short from the more volatile parts of pragmatics and must leave them well alone. On the other hand, important pragmatic concepts like "approximate equality", although apparently unmathematical, have turned out not to be among those volatile parts, as indicated by Concurrency Theory.

Literature

1. Petri, C.A.: *Grundsätzliches zur Beschreibung diskreter Prozesse*. 3. Colloquium über Automatentheorie. Basel: Birkhäuser Verlag, pp. 121-140 (1967) (*Engl. translation available from GMD*)
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4. Petri, C.A.: *Introduction to General Net Theory*. Lecture Notes in Computer Science Vol. 84: Net Theory and Applications, Proc. of the Advanced Course on General Net Theory of Processes and Systems, Hamburg, 1979 / Brauer, W. (ed.). Berlin-Heidelberg-New York: Springer-Verlag, pp. 1-19 (1980)
5. Petri, C.A.: *Concurrency Theory*. In: Springer Lecture Notes in Computer Science (Advanced Course in Petri Nets-1986), Springer-Verlag (1987)

A general survey on the methods of net theory as well as an extensive literature list can be found in:

6. Genrich, H.J.; Lautenbach, K.; Thiagarajan, P.S.: *Elements of General Net Theory*. Lecture Notes in Computer Science Vol.84: (see [4] above) pp. 21-163 (1980)