Some Simple Extensions of Petri's Cycloids

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Contents

Introduction

Introduction

- Cycloids are special nets with cyclic behaviour:
  The main idea is to fold a special subnet of an infinite causal net (Petri space) in such a way that the behaviour of this initially marked subnet (cycloid) can be repeated infinitely often without losing or gaining any subnet behaviour and to stay bounded.
- Already considered by Petri himself
- Kummer and Stehr (1997) and Valk (2018, 2019, 2020) conduct further work

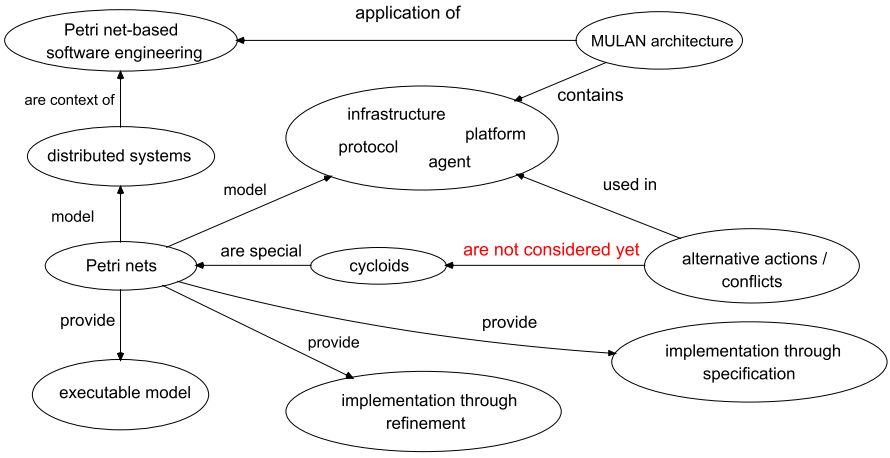
Objective and Context

Objective and Context

- Cycloids contain concurrency, but no conflicts (alternative actions).
- We consider the modelling of conflicts in cycloids as an extension[1]
- We give a context of Petri net-based software engineering.

[1] some more basic and more formal results will be discussed in the talk of Valk at the main Petri net conference.

Objective and Context



Cycloids

Cycloids

Cycloids model recurrent behaviour containing causal dependencies and concurrency. Formally, they are foldings of an infinite Petri net (*Petri space*).

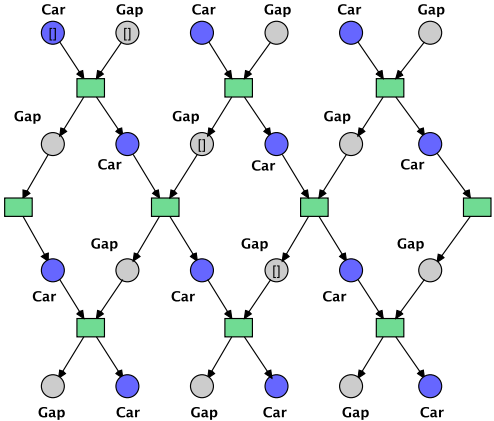
→ Four parameters specify how the infinite Petri space is folded.

→ Different cycloids can be generated / chosen from the same Petri Space. The choice represents the desired modelling perspective of the modellers.

Cycloids

Infinite model of driving cars
(Space horizontal, time vertical)

Cars blue, gaps grey
Depiction on the right hand side shows excerpt of the following Petri space:



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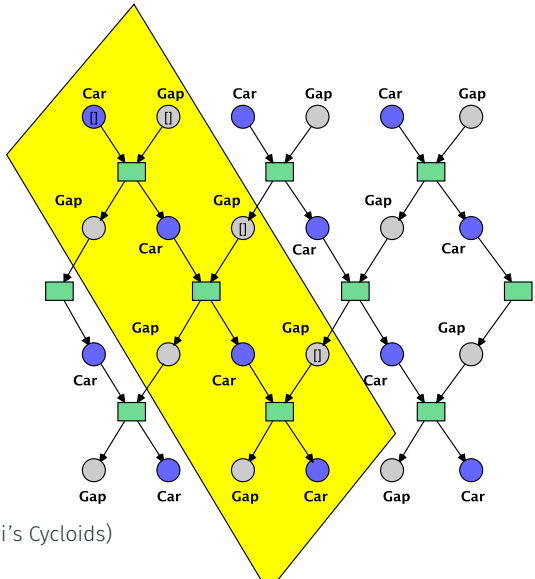


Cycloids

The *Petri space* is a Petri net $PR = (P_r, T_r, F_r, W_r)$, which is defined as follows:

$P_r = P_r^{\rightarrow} \cup P_r^{\leftarrow}$, with $P_r^{\rightarrow} = \{p_{\xi,\eta}^{\rightarrow} | \xi, \eta \in \mathbb{Z}\}$ and $P_r^{\leftarrow} = \{p_{\xi,\eta}^{\leftarrow} \mid \xi, \eta \in \mathbb{Z}\}, P_r^{\rightarrow} \cap P_r^{\leftarrow} = \emptyset$,

$T_r = \{t_{\xi,\eta} \mid \xi, \eta \in \mathbb{Z}\}$,

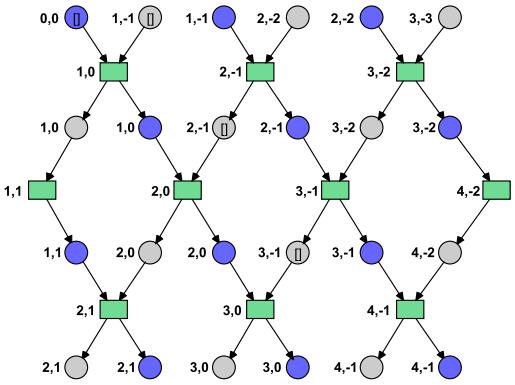
$F_r = \{(t_{\xi,\eta}, p_{\xi,\eta}^{\rightarrow}) \mid \xi, \eta \in \mathbb{Z}\}$

$\cup \{(p_{\xi,\eta}^{\rightarrow}, t_{\xi+1,\eta}) \mid \xi, \eta \in \mathbb{Z}\}$

$\cup \{(t_{\xi,\eta}, p_{\xi,\eta}^{\leftarrow}) \mid \xi, \eta \in \mathbb{Z}\}$

$\cup \{(p_{\xi,\eta}^{\leftarrow}, t_{\xi,\eta+1}) \mid \xi, \eta \in \mathbb{Z}\}$,

$W_r(x, y) = 1, \forall (x, y) \in F_r$.



Cycloids

For $\alpha, \beta, \gamma, \delta \in \mathbb{N} \setminus \{0\}$ we define an equivalence relation $\equiv$ on X_r for $X_r = P_r \cup T_r$:

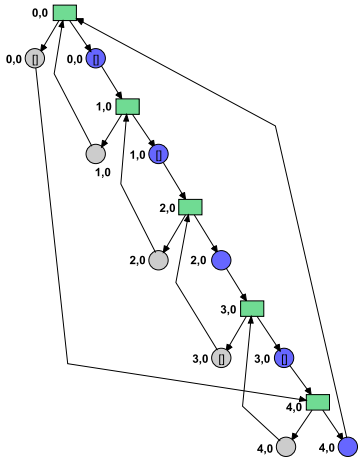
$\equiv[P_r^{\rightarrow}] \subseteq P_r^{\rightarrow}$,

$\equiv[P_r^{\leftarrow}] \subseteq P_r^{\leftarrow}$,

$\equiv[T_r] \subseteq T_r$,

$x_{\xi,\eta} \equiv x_{\xi+m\alpha+n\gamma,\eta-m\beta+n\delta}, \forall \xi, \eta, m, n \in \mathbb{Z}$.

Example: $\mathcal{C}(2, 3, 1, 1)$



Cycloids

Then a particular net (P, T, F, W) is defined as a cycloid $\mathcal{C}(\alpha, \beta, \gamma, \delta)$:

P contains the places of $X_r/_{\equiv}$,

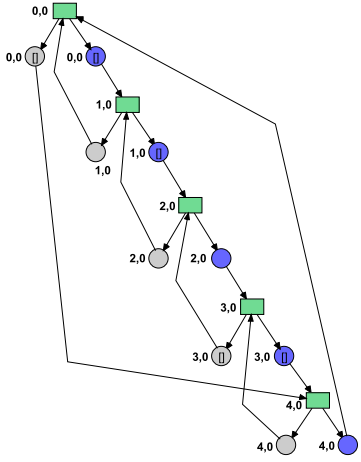
T contains the transitions of $X_r/_{\equiv}$,

$([x]_{\equiv}, [y]_{\equiv}) \in F \Leftrightarrow \exists x' \in [x]_{\equiv} \exists y' \in [y]_{\equiv} : (x', y') \in F_r$,

$W(x, y) = 1, \forall (x, y) \in F$.

Furthermore α and β specify a standard initial marking for the cycloid. The maximal distance between cars is restricted by the number of available spaces ($cars + gaps = spaces$).

Example: $\mathcal{C}(2, 3, 1, 1)$



Extensions

Overview

	Model	Focus on
•	m cars, n gaps, one Lane	concurrency
•	multiple independent lanes, no crossing of lanes	concurrency
•	additional overtaking bay of length one, one car overtakes m cars	conflict
•	additional overtaking lane of length l, one car overtakes m cars	conflict
•	guaranteed cut in	conflict
•	forced sheering out	conflict
•	guaranteed cut in + forced sheering out	conflict
•	multiple cars can overtake	conflict
•	multiple cars must overtake	conflict

Overview

	Model	Focus on
•	multiple cars are located on one section	coarsening
•	sliding window of overtaking	conflict
•	crossing lanes freely	conflict
•	coloured cars	distinguishability
•	coloured cars + coloured gaps	distinguishability
•	oncoming traffic on additional lane	conflict
•	simultaneous actions of cars	synchronization

Additional Lane

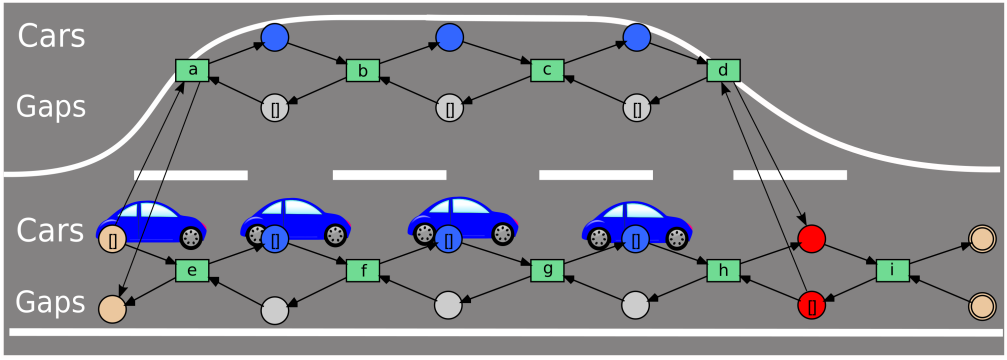


Figure 1: Additional Lane

Beige places are modelled as virtual places for cars and gaps respectively.

Additional Lane

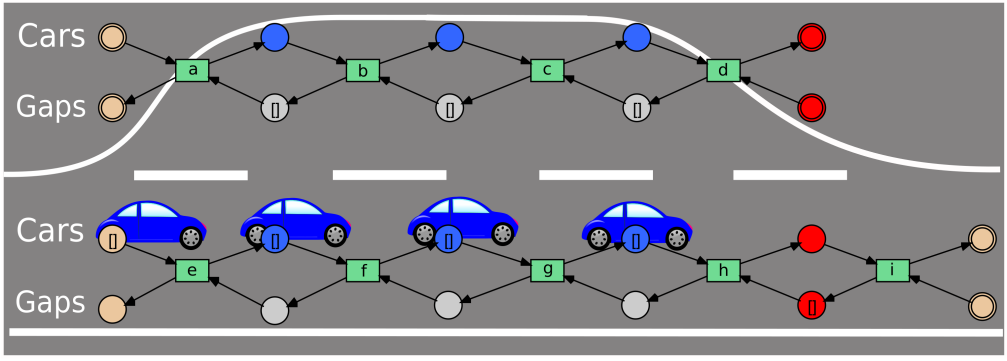


Figure 2: Additional Lane

Beige and red places are ”copied” for cars and gaps respectively.

Crossing of Lanes

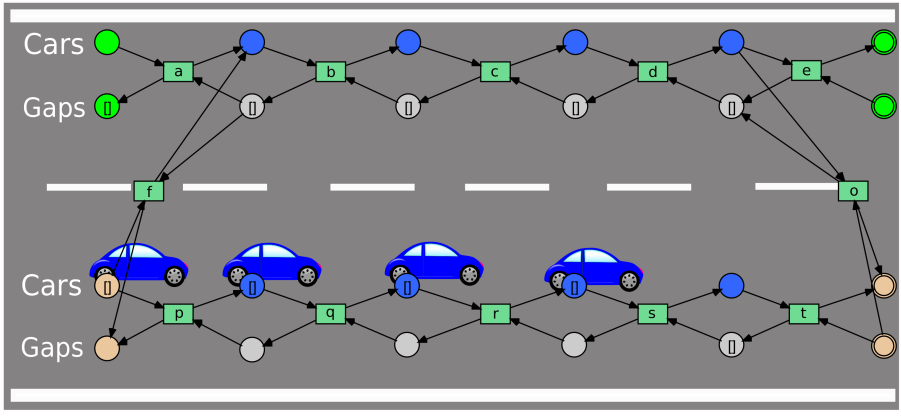


Figure 3: Crossing of Lanes

Beige and green places are ”copied” for cars and gaps respectively.

Crossing of Lanes

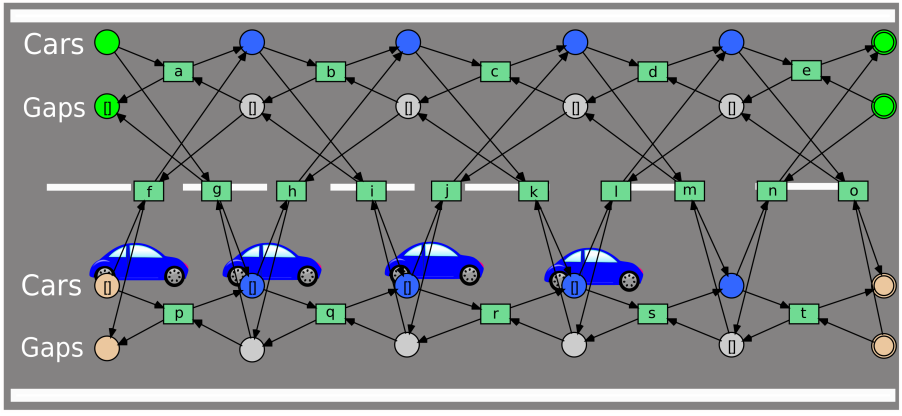


Figure 4: Crossing of Lanes

Beige and green places are ”copied” for cars and gaps respectively.

Guaranteed Cut In

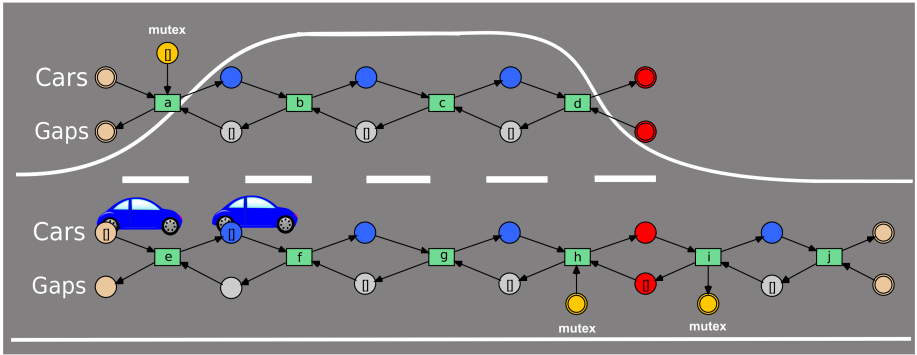


Figure 5: Guaranteed Cut In

Beige and red places are ”copied” for cars and gaps respectively.

Application: autonomously driving cars

Forced Sheering Out

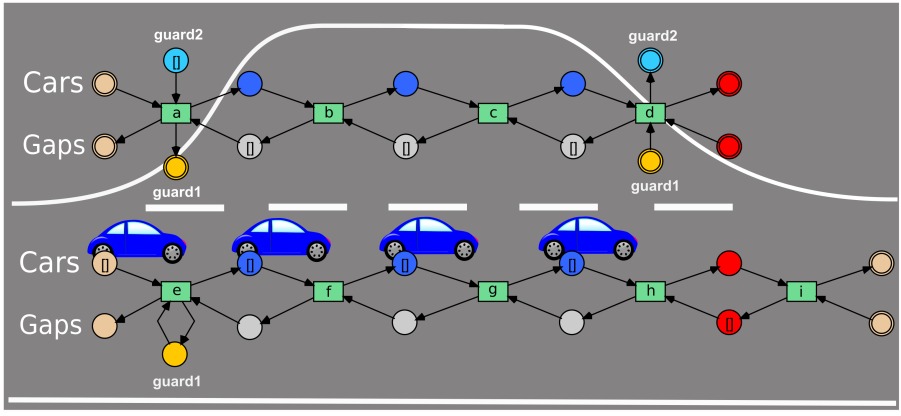


Figure 6: Forced Sheering Out

Beige and red places are ”copied” for cars and gaps respectively.

Synchronization

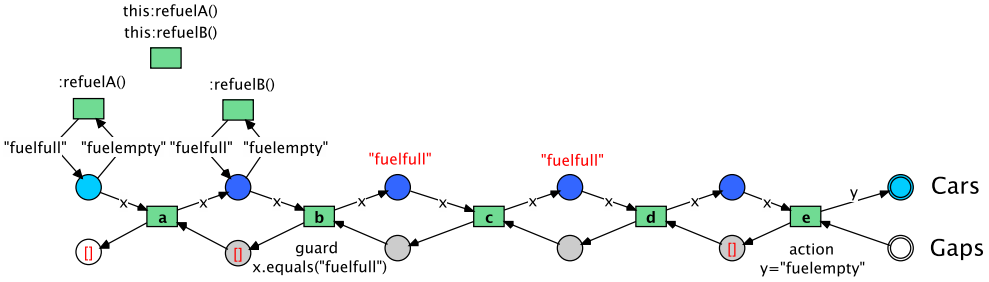


Figure 7: Synchronization

Light blue and white places are ”copied” for cars and gaps.

Applications for (Extended) Cycloids

Modelling

Relevant modelling aspects of cycloids are:

- true concurrency
- no conflicts or other extensions
- simple structure
- difficult to represent
- require tool support
- low expressiveness
- formal background
- applicable for basic modelling

Modelling

Relevant modelling aspects of extended cycloids are:

- true concurrency
- conflict place / virtual places / colours / synchronous channels etc.
- still simple structure
- even more difficult to represent
- require tool support
- increased expressiveness
- no formal background (yet)
- more applicable for modelling

Mapping of Entities to Extended Cycloids

Currently envisioned modelling areas

- basis for understanding concurrency
- object behaviour
- protocols between agents
- incremental scenario modelling support
- proof by construction principle in software engineering
- secure systems

Mapping of Entities to Extended Cycloids

What are the benefits of conflicts for cycloids?

- single cycloid: no alternative for leaving out behaviour, just contacts hinder tokens to move forward
- two cycloids: independent behaviour of two cycloids
- High redundancy when only two transitions differ in the set of transitions T
- solution: alternative parts of the cycloid
- general assumption: repetitive behaviour

Mapping of Entities to Extended Cycloids

Construction methods for extended cycloids

- replication (colouring)
- composition
- folding
- fusion
- synchronization

Mapping of Entities to Extended Cycloids

Generalisation:
A Unit or an entity has some behaviour creating a lifeline (Minkowski/Petri).

Idea:
Units are identified within the Petri space and modelled by (extended) cycloids.

Units are combined to units with the above mentioned methods.

Modellers identify the relevant units of a system.

Software Engineer can follow the proof-by-construction principle for secure systems.

Summary and Future Work

Summary

- Cycloids as special nets
- Options for modelling of recurrent behaviour described
- Conceptual extensions of cycloids are sketched
- General modelling paradigm for secure systems proposed

Future Work

- Tools for the generation of processes of nets
- Investigating formal properties of extended cycloids
- Modelling approach for provable systems based on proof by construction principles
- Tools for the support of modelling, simulation and analysis of extended cycloid models

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