HEBBIAN SPIKE-TIMING DEPENDENT SELF-ORGANIZATION IN PULSED NEURAL NETWORKS

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Abstract

We present a mechanism of unsupervised competitive learning and development of topology preserving self-organizing maps of spiking neurons. The information encoding is based on the precise timing of single spike events. The work provides a competitive learning algorithm that is based on the relative timing of the pre- and post-synaptic spikes, local synapse competitions within a single neuron and global competition via lateral connections. Furthermore, we present part of the experimental work on the capability of the suggested mechanism to perform topology preserving mapping and competitive learning. The results show that our model covers the main characteristic behaviour of the standard SOM but uses a computationally more powerful timing-dependent spike encoding.

1 Introduction

There is an ongoing debate and research on which are the essential properties of the biological neurons that need to be simulated in order to reach the computational power of a real neural system [Maass, 1997b]. As a result of some recent studies, there is a broad agreement that the brain uses simultaneously both mean firing rate as well as spike-timing encoding schemes in order to represent information and transfer signals [Gerstner and van Hemmen, 1994, Sejnowski, 1995]. Artificial neural networks of spiking neurons which employ the mechanism of precise spike timing for encoding information have been shown to be computationally more powerful than the classical connectionist models [Maass, 1997b]. Furthermore, there are several temporal encoding schemes that the real neurons are believed to be using. Neural formations such as cell assemblies rely mainly on the synchronization and coincidence of the spikes, whereas encoding schemes such as phase and time-to-first spike codes encode signals in the time shift between the spikes or with a reference to a global oscillation. In this paper we present a model of an artificial neural network of spiking neurons that uses the time shift of the incoming spikes with a reference to a global signal as a mechanism of encoding information.

Topology preserving maps have been found in many areas in the brain [Arbib, 1995] and are believed to emerge as a result of a competitive learning mechanism [Miller, 1996, Sirosh and Miikkulainen, 1996]. The Self-Organizing Map (SOM) architecture has provided a good explanation and computational models of the mechanism of developing such topological maps [Kohonen, 1993]. There has been a fair amount of realizations of the basic idea of self-organization in artificial neural networks, most of which are based on the connectionist sigmoidal gates. Such models, however, could not exploit the computational advantages provided by the neurons using temporal codes.

2 Neuron model, Pulsed Neural Network and Information encoding

We used a leaky integrate-and-fire neuron described in [Maass, 1999] with post-synaptic and soma potentials modeled with differential equations. In the different experiments, we varied the time constants of the synapse and the soma so that the neuron reaches the peak of its membrane potential at time $\hat{t}$ between 5 and 20 milliseconds. The neuron also fires exactly at time $\hat{t}$ if the total synaptic strength of simultaneous spikes equals 1.

The network consists of a layer of $m+1$ input neurons $u_0, u_1, \ldots, u_m$, and a layer of n competitive neurons $v_1, \ldots, v_n$ (Figure 1). A competitive neuron v receives excitatory signals from the input neurons via feed-forward connections with normalized weights $w_0, w_1, \ldots, w_1$. The competitive neurons are connected to each other via lateral connections which are initially excitatory for neighboring neurons and inhibitory for neurons on large distances in the neighborhood matrix.

The input information is encoded in the latency of the spikes at $u_1, \ldots, u_m$ with respect to a reference to a “global” signal at time t_0. In our model, the reference signal is given by a single spike of u_0 at t_0. In order to represent an input vector $(x_1, \ldots, x_m)$, each u_i fires exactly once within the encoding time interval T after t_0 representing a value $x_i = \frac{t_i - t_0}{T}$.

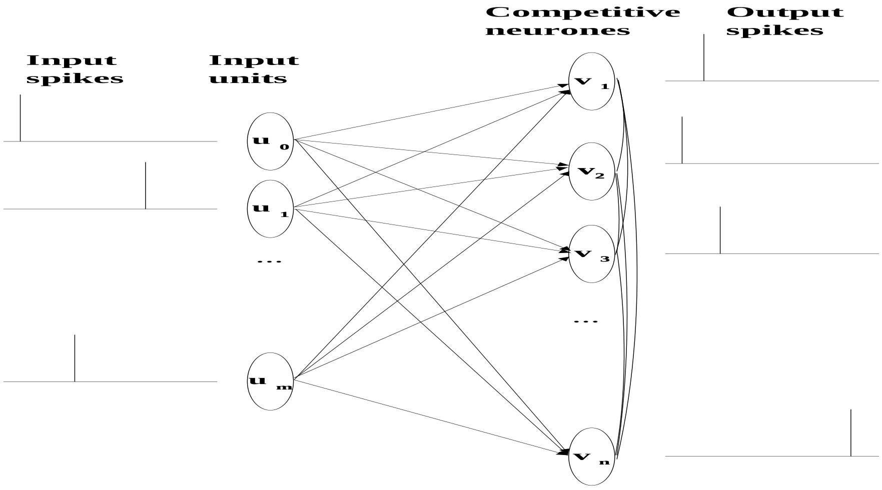

Figure 1: The neural network architecture.

3 Hebbian Spike-Timing Dependent Self-Organization

The paradigm of correlation-based (Hebbian) learning provides the ground for the current views on the development of neural circuits on the basis of correlated activity. Such learning is known to run on two critical mechanisms: activity-dependent synaptic modification as suggested by Hebb [Hebb, 1949] and local synaptic competition and strength redistribution [Miller, 1996]. Both mechanisms are equally important and found to be active in the self-organization and competitive adaptation of real neural systems. In addition, a self-organization of neurons needs a mechanism of global competition between the neurons believed to be expressed as a mutual inhibition between the competitive nodes.

Let us consider a set X of m-dimensional input vectors and $x^p = (x_1^p, \dots, x_m^p)$ presented as an input to a network with $m+1$ inputs and n competitive neurons. A competitive neuron v_i receives an excitatory feed-forward signal from each of the input neurons u_j with weight w_{ij} at time corresponding to the value of x_j^p. If we assume that neuron v_k is the first competitive neuron to fire (at time t_k) as a response to the input, the nodes $v_i(for i \neq k)$ will receive a signal from the lateral connections with strength $\tilde{w}_{ik}$. If the lateral connections are excitatory for the neurons that are topologically close and inhibitory for the remote neurons, then the “winner” will drive the firing times of the neighboring neurons towards t_k. Respectively v_k will delay or prevent remote neurons from firing. Our goal in such a situation is to adjust the weights of v_k so that it becomes the fastest neuron that could possibly fire to input spikes encoded in x_p, as well as the weights of the neighboring neurons, so that they will be faster to respond to inputs similar to x_p.

In discrete simulations with exhaustive search, we obtained the optimal weights of neurons with two and three synapses. For each particular interval between the pre- and post-synaptic spikes $\Delta t = t_{pre} - t_{post}$, we explored the sets of possible normalized synaptic weights and recorded the one which leads to the fastest response of the neuron. Plotted against Δt, the optimal weights formed a curve (Figure 2) which we approximate as function $\mathscr{F}(\Delta t)$. The adaptation rule suggested here iteratively adapts the weights of the competitive neurons towards respective values from $\mathscr{F}(\cdot)$. If after the response to a given input, we shift the weights of the competitive neurons that have fired towards the optimal weights with respect to their delay in the response, we will effectively decrease their response times to the same input, for the winner, or to similar inputs, for the neurons topologically close to the neuron.

$$\Delta w_{ij} = \eta(\mathscr{F}(t_{pre} - t_{post}) - w_{ij}) \tag{1}$$

where $\mathscr{F}(\cdot)$ is the optimal weights function of the difference between the pre-synaptic ((t_{pre}) and post-synaptic (t_{post}) spike times, and η is the learning rate.

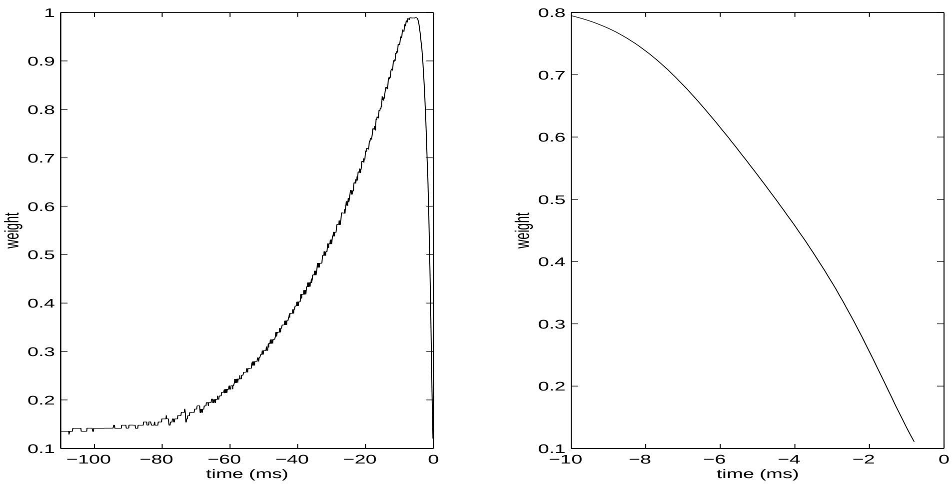

Figure 2: Optimal weights as a function of the relative timing between the pre- and post-synaptic spikes ($t_{pre} - t_{post}$). Left: the actual values of the optimal weights obtained for a neurons with two synapses. Right: Exponential approximation from the values of the optimal weights obtained for a neuron with three synapses.

The local synaptic competition is achieved via intrinsic quasi normalization of the weights vectors. Since $\mathscr{F}(\cdot)$ describes normalized weights, after training the weights vector of a single neuron is very close to normalized.

4 Simulations

We tested our model with two of the standard examples of self-organization in artificial neural networks, i.e. one- and two-dimensional patterns. In the first experiment, we used 10 one-dimensional patterns uniformly distributed in $[0,1]$ which where presented to a network with 2 input and 10 competitive neurons. The lateral weights where decreasing with the distance between the neurons, starting with slightly positive for the first and second neighbors, and running negative afterwards. The learning was based on the optimal weights function for two weights as is shown on the left in Figure 2. The results from the simulation are shown in Figure 3. After about 1000 learning epochs, the network has a topology preserving map of the input set. Due to the relationship between the neural dynamics and the weights, the relationship between the firing times and input patterns in not exactly linear, but rather follows the behaviour of the optimal weights curve and the form of the membrane potential.

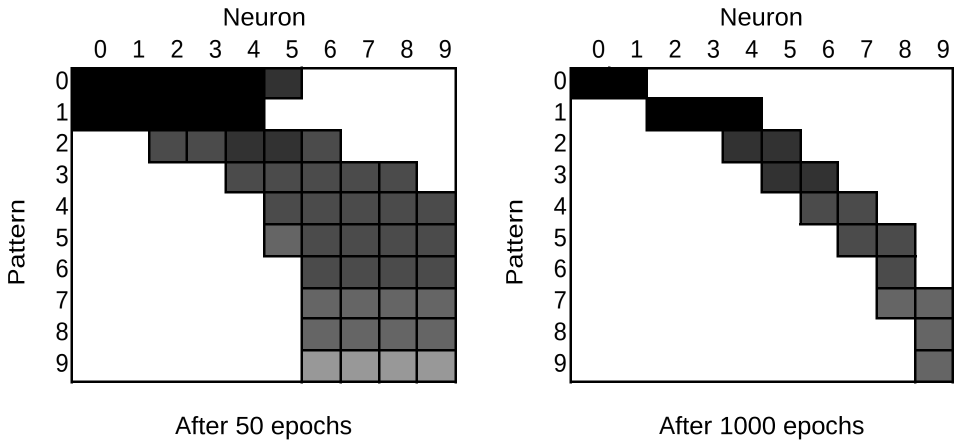

Figure 3: Competitive learning of one-dimensional input. Each field represents the firing time of a neuron to a particular pattern. The darker the color, the faster the response.

In the second experiment, we used two-dimensional patterns uniformly distributed in a square region [0,4] $\times$ [0,4]. The goal was to organize the competitive neurons in a topology preserving grid. The network consisted of two input and 5$\times$5 competitive neurons. The lateral weights were initialized as in the previous experiment. In both, simulations the lateral weights and the learning rate were decreased after each epoch. Results from the second experiment are shown in Figure 4. Since the relationship between the weights and the input vectors is not linear, the usual direct comparison of the weights and the input patters is not applicable it this case. We present three examples of a typical response of the network to the input patterns (0,0), (0,4) and (4,4).

As it can be seen in both experiments, the learning algorithm was able to achieve a topology-preserving SOM of the competitive neurons. For each input signal, there was a single competitive neuron responding first, and in some cases some of its immediate neighbors fired with a small delay.

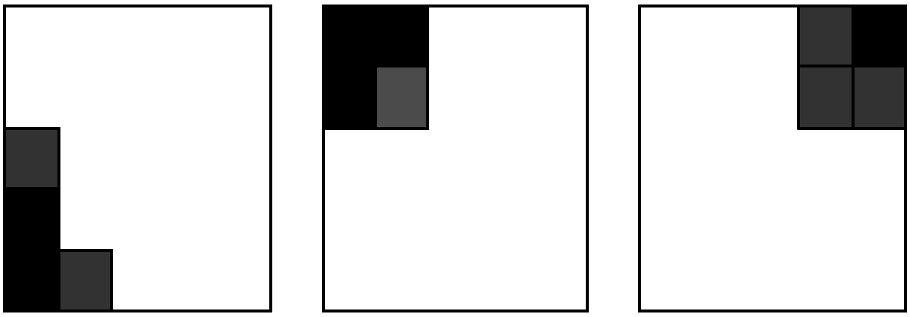

Figure 4: Competitive learning of two-dimensional input. Each field represents the firing time of a neuron to a particular pattern. The darker the color, the faster the response.

5 Discussion

A few computational models and competitive learning mechanisms that have been built upon networks of spiking neurons have been suggested. A model of self-organizing maps of spiking neurons has been applied in computational modeling of the pattern interaction and orientation maps in the primary visual cortex [Choe and Miikulainen, 1998, Sirosh and Miikkulainen, 1997]. However, this model explores only the firing rate of the spiking neurons and learning occurs only after the network has reached a stable state of firing.

A mechanism of competitive learning and self-organization in networks of spiking neurons has also been presented in [Ruf and Schmitt, 1998]. The work is based on a spiking neuron model presented in [Maass, 1997a] and implements a mechanism of encoding the signal in the precise timing of the spikes. Experimentally, the model was shown to exhibit the same characteristic behaviour as the standard topology preserving SOM. The learning algorithm is built under the simplifying assumptions of a linear rising phase of the post-synaptic potential. Although later it has been shown that the algorithm can be applied in the case of a non-linear post-synaptic response, it is not clear and our experiments show that it might not be the case that the learning suggested in [Ruf and Schmitt, 1998] will still lead to optimal weights in the network.

Another idea of competitive learning with spiking neurons is presented in [Song et al., 2000]. The research is based on experimental and modeling studies and suggest a form of spike-timing dependent synaptic plasticity based on the relative timing of pre- and post-synaptic spikes. The work explores the role of such plasticity to facilitate the irregular but more sensitive to pre-synaptic spike timing firing of the neuron and also concentrates on the local competition between the synapses. Although the overall behaviour of the model presented there is close to our results, there is one main difference with our mechanism. It is expressed in the learning window for the relative time of the pre- and post-synaptic spikes close to zero, where in contrast to the large strengthening of the synapse suggested in [Song et al., 2000], our results show that some weakening should occur.

The results presented in this paper show that our model covers the main characteristic behaviour of real self-organizing maps [2]. The suggested approach allows the competitive behaviour of the real neurons to be explored in a simulation with computationally more powerful and biologically plausible models of spiking neurons processing temporally encoded information. The derived learning follows experimental results on synaptic plasticity in real neurons and presents some new insight into the changes which occur when the timings of the pre- and post-synaptic spikes are very close in time. The research presented here demonstrates the advantages of a bi-directional modeling approach between computer science and neuroscience. It derives ideas from our current knowledge of how the real neurons operate in order to develop a novel computational model for a learning algorithm in spiking neurons. Furthermore, the results from simulations with this algorithm are used to develop hypotheses about the synaptic plasticity and self-organization of the real neurons.

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