



FIAS Frankfurt Institute  
for Advanced Studies



# Learning Coordinated Eye and Head Movements: Unifying Principles and Architectures

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# Coodinated Eye/Head Movements



- ❖ Saccade: Quick change of gaze direction
- ❖ Multi-segment control of different motor systems
- ❖ Saccades are ballistic movements (Leung et al. 2008)
- ❖ Head-restrained vs. Head-free

# Open Questions



Do saccade kinematics result from some optimality principle?

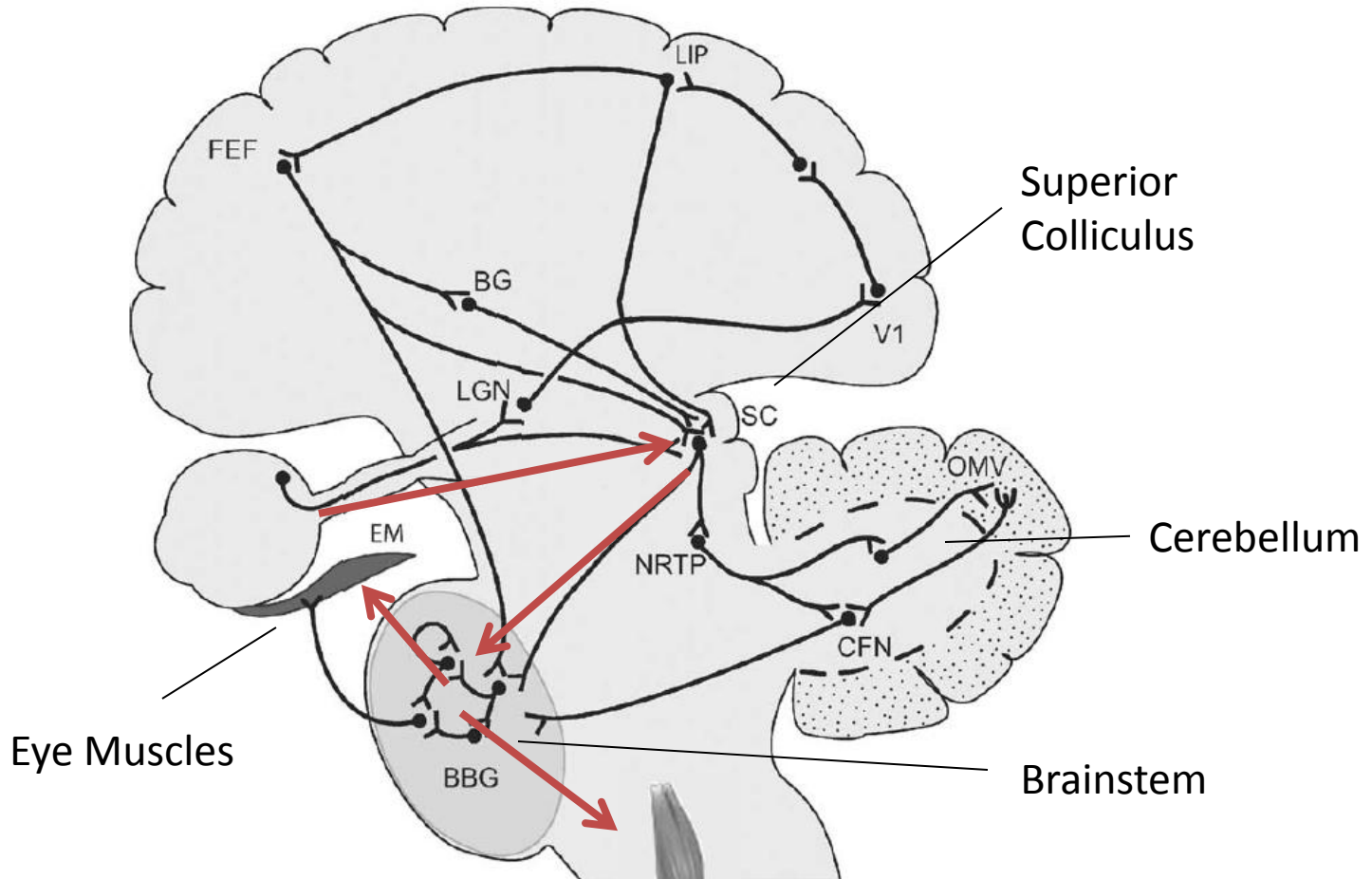


How does the brain learn saccade kinematics?



Is it possible to address both questions above using a single computational model?

# Biological Basis of Saccadic Eye/Head Movements



*Hopp & Fuchs (2004)*

# Eye/Head Plant Models

➔ **Eye Plant:** Linear models sufficient for realistic saccades (Van Opstal et al, 1985)

$$T_1 T_2 T_3 \ddot{r}_e(t) + (T_1 T_2 + T_2 T_3 + T_3 T_1) \dot{r}_e(t) + (T_1 + T_2 + T_3) r_e(t) = g_e u_e(t)$$

$$h_e(t) = g_e (k_1 e^{-\frac{t}{T_1}} + k_2 e^{-\frac{t}{T_2}} + k_3 e^{-\frac{t}{T_3}})$$

$$r_e(t) = \int_0^t u_e(\tau) h_e(t - \tau) d\tau$$

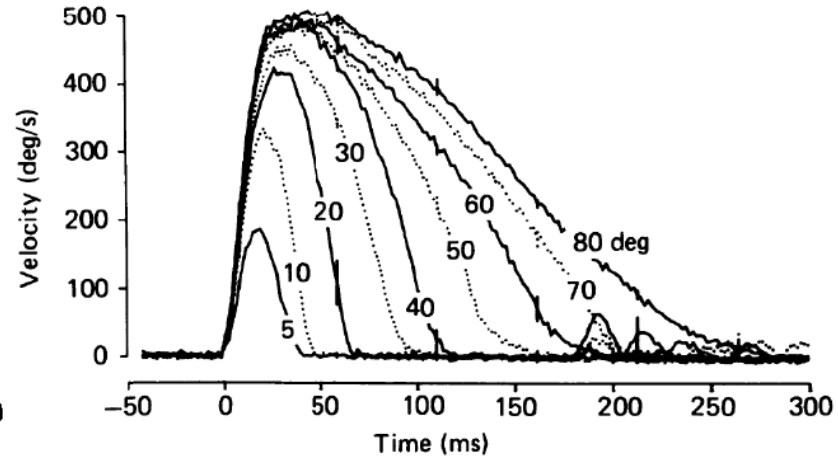
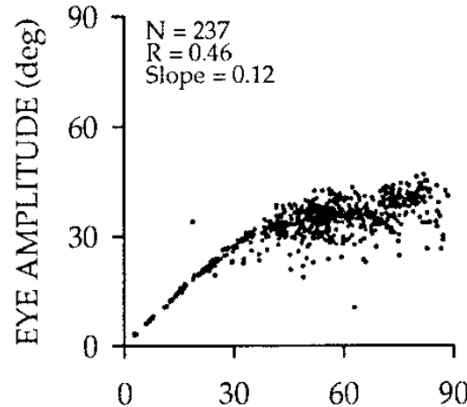
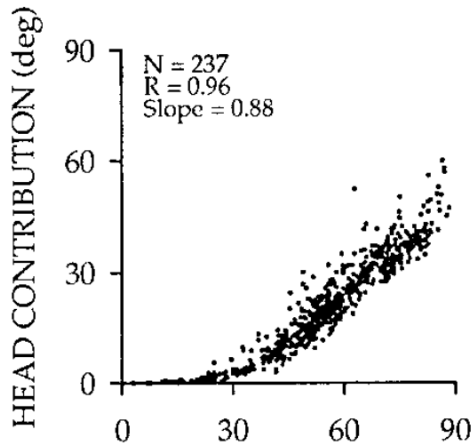
➔ **Head Plant**

$$T_4 T_5 \ddot{r}_h(t) + (T_4 + T_5) \dot{r}_h(t) + r_h(t) = g_h u_h(t)$$

$$h_h(t) = g_h (k_4 e^{-\frac{t}{T_4}} + k_5 e^{-\frac{t}{T_5}})$$

$$r_h(t) = \int_0^t u_h(\tau) h_h(t - \tau) d\tau$$

# The Neural Control Signal: Optimization



$$u_e(t) = ? \longrightarrow \text{Eye Plant}$$

$$u_h(t) = ? \longrightarrow \text{Head Plant}$$

- Can we derive the control signals from some optimality principle?
- How are the control signals optimized through learning?

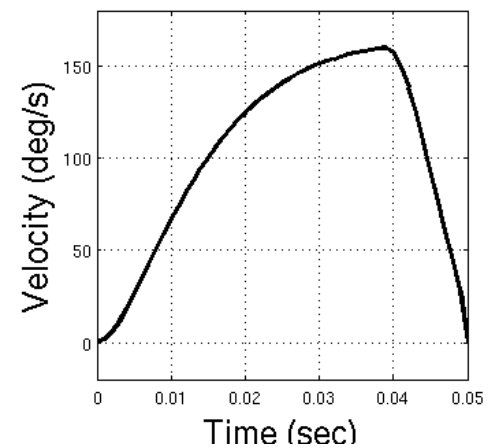
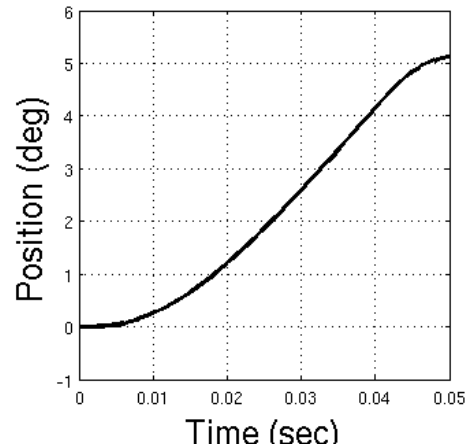
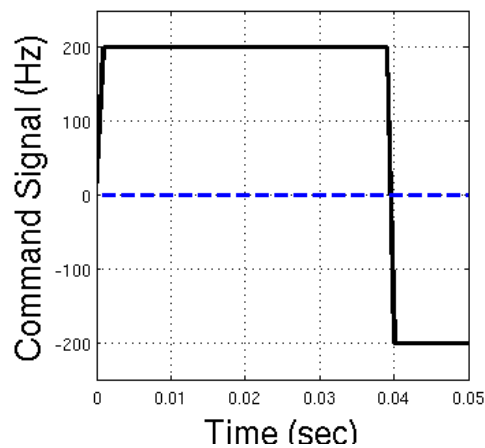
# Optimality Principles



- No proper vision during saccades due to **motion blur** or **neural suppression**.

## ➤ Minimum Time

The control signal instantaneously switches between its maximum positive and negative values to accelerate and decelerate the eye (“bang-bang”).



# Models: Optimality Principles

- **Minimum Variance:** (*Harris & Wolpert, 1998; 2006*)

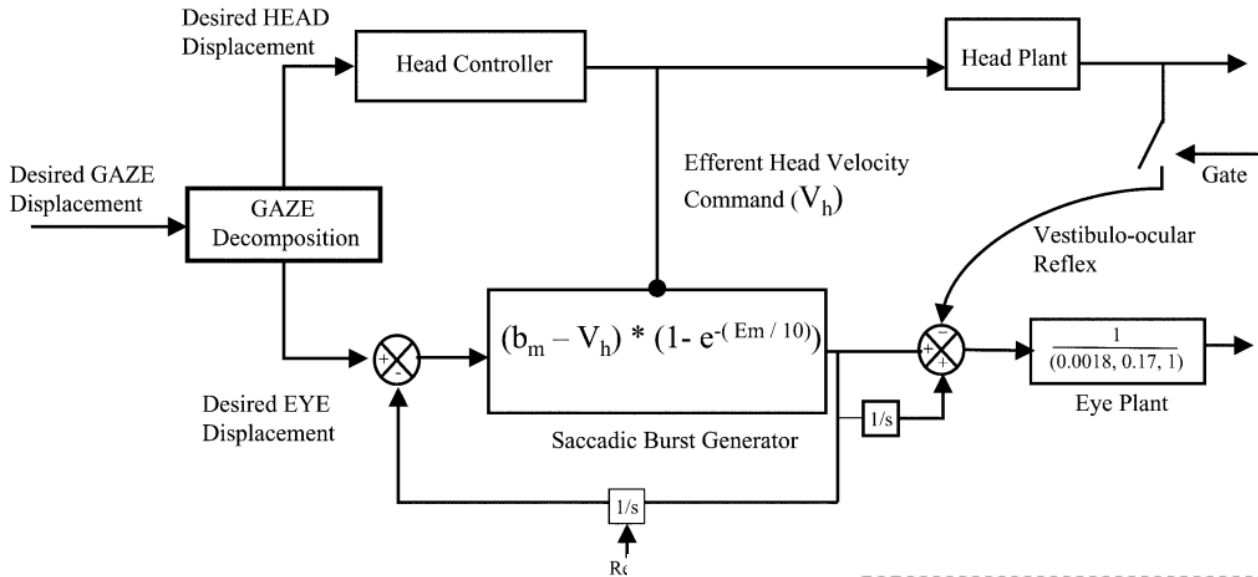
$$\text{Cost} = \underbrace{\int_0^T \alpha dt}_{\text{Movement cost}} + \underbrace{\int_T^{T+F} \beta e(t)^2 dt}_{\text{Fixation cost}} \quad \sigma_u(t) = k|u(t)|$$

- **Minimum Effort** (*Kardamakis & Moschovakis, 2009*)

$$\text{Cost} = \int_0^T [\alpha(x_e)u_e^2 + \beta u_h^2] dt$$

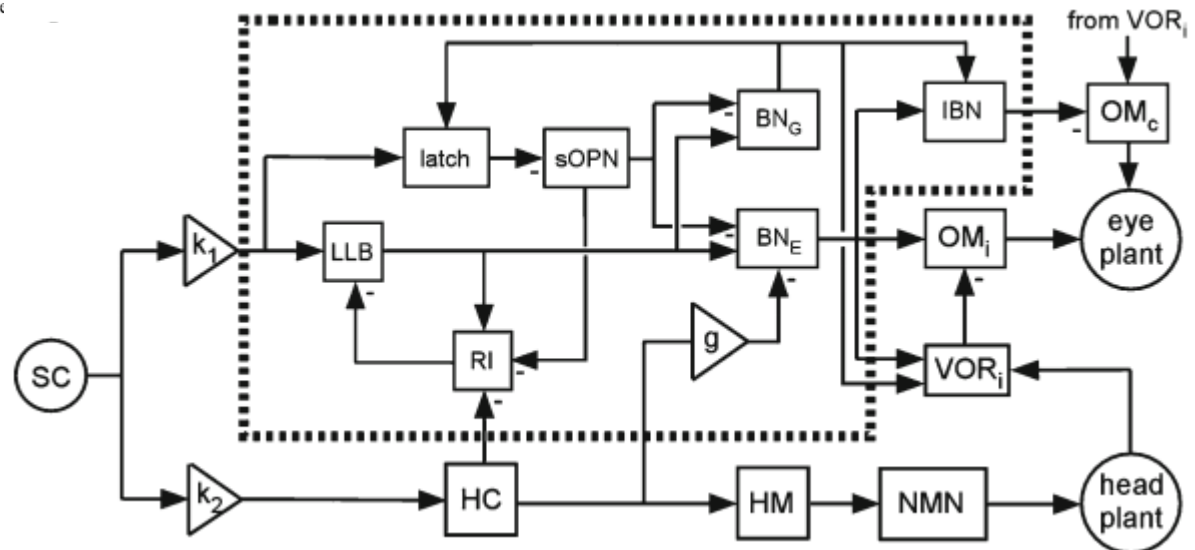
$$\alpha(x_e) = \alpha_0 + \alpha_1 x_e + \alpha_2 x_e^2$$

# Models: Neural Architectures

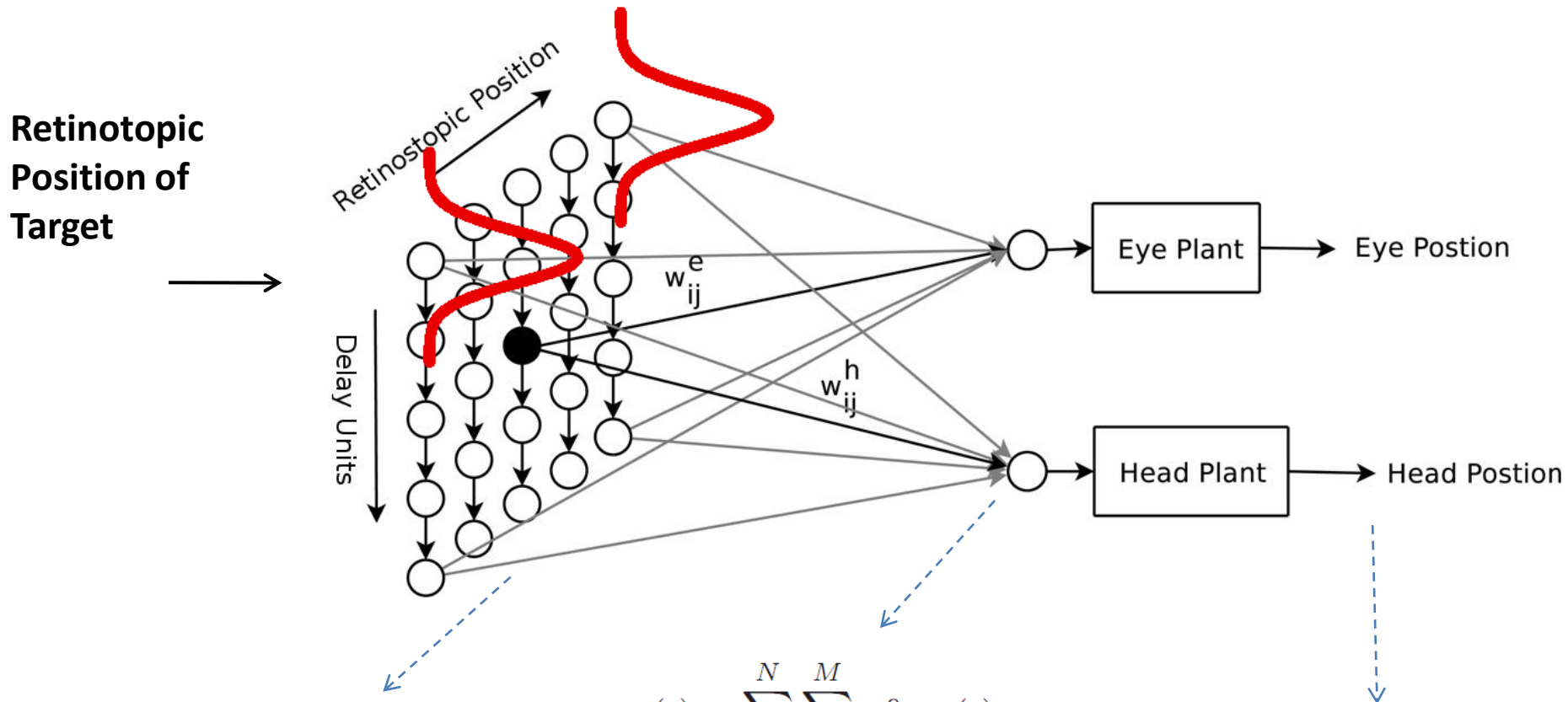


*Freedman (2001)*

*Kardamakis et al. (2010)*



# The Open-loop Neural Controller



$$s_{ij}(t) = A \exp\left(-\frac{(i - \frac{t}{\Delta t})^2}{2\sigma^2}\right)$$

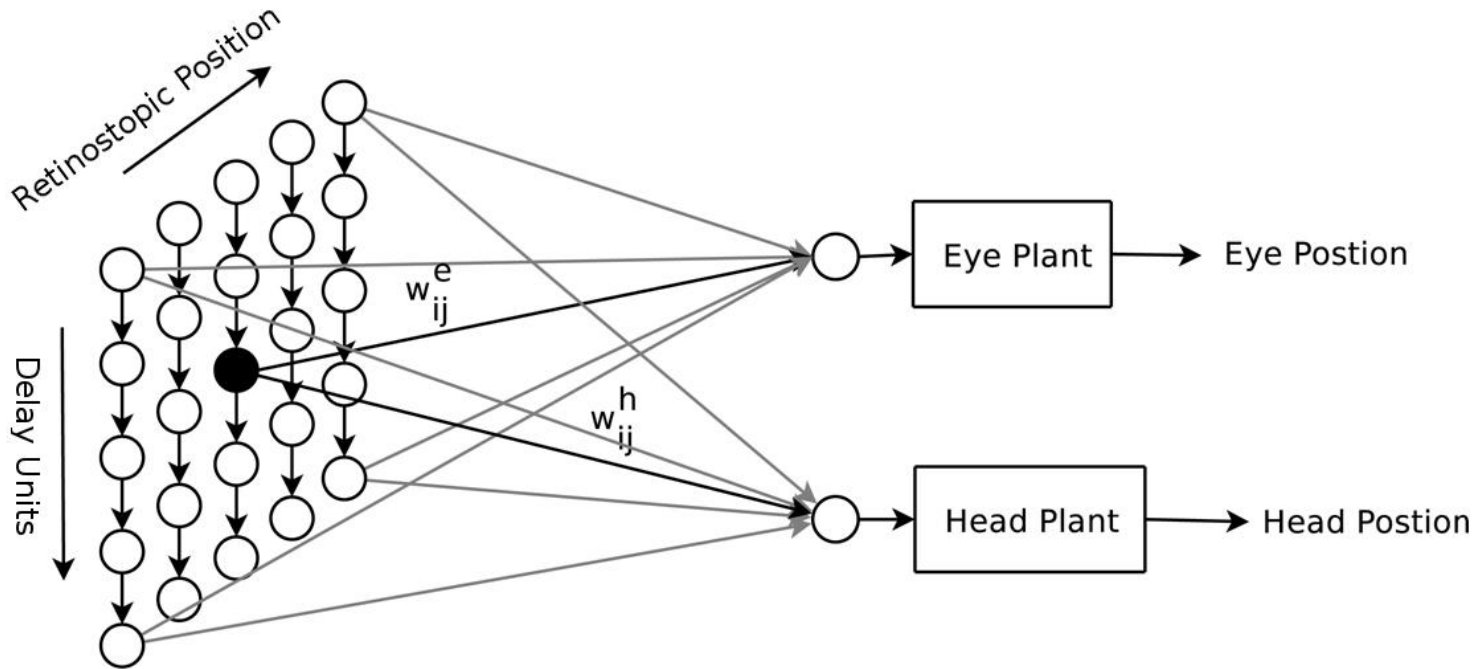
$$u_e(t) = \sum_{j=1}^N \sum_{i=1}^M w_{ij}^e s_{ij}(t)$$

$$u_h(t) = \sum_{j=1}^N \sum_{i=1}^M w_{ij}^h s_{ij}(t)$$

$$r_e(t) = \int_0^t u_e(\tau) h_e(t - \tau) d\tau,$$

$$r_h(t) = \int_0^t u_h(\tau) h_h(t - \tau) d\tau$$

# The Cost Function



1. The gaze should reach the target as soon as possible and then stand still on the target position.
2. The power of the neural command signal should be constrained.

$$E = \int_0^T |r_{\text{vis}}(t)| dt + \sum_{j=1}^N \sum_{i=1}^M (\alpha_e |w_{ij}^e|^n + \alpha_h |w_{ij}^h|^n)$$

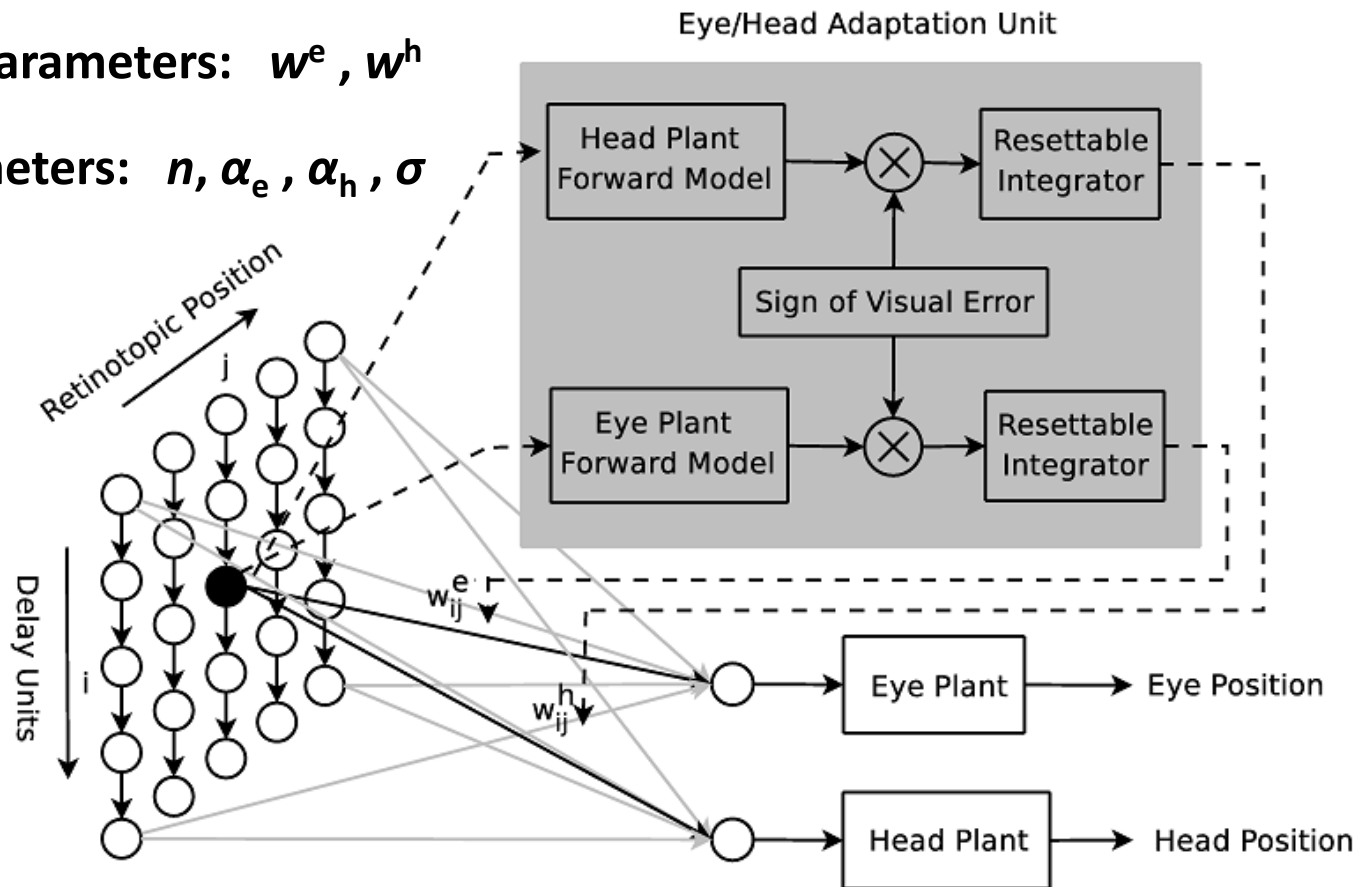
# Adaptation: Gradient Descent on the Cost Function

$$\Delta w_{ij}^e = \delta_{ij}^e \int_0^T \text{sign}(r_{\text{vis}}(t)) \left( \int_0^t s_{ij}(\tau) h_e(t - \tau) d\tau \right) dt - n \delta_{ij}^e \alpha_e (w_{ij}^e)^{(n-1)}$$

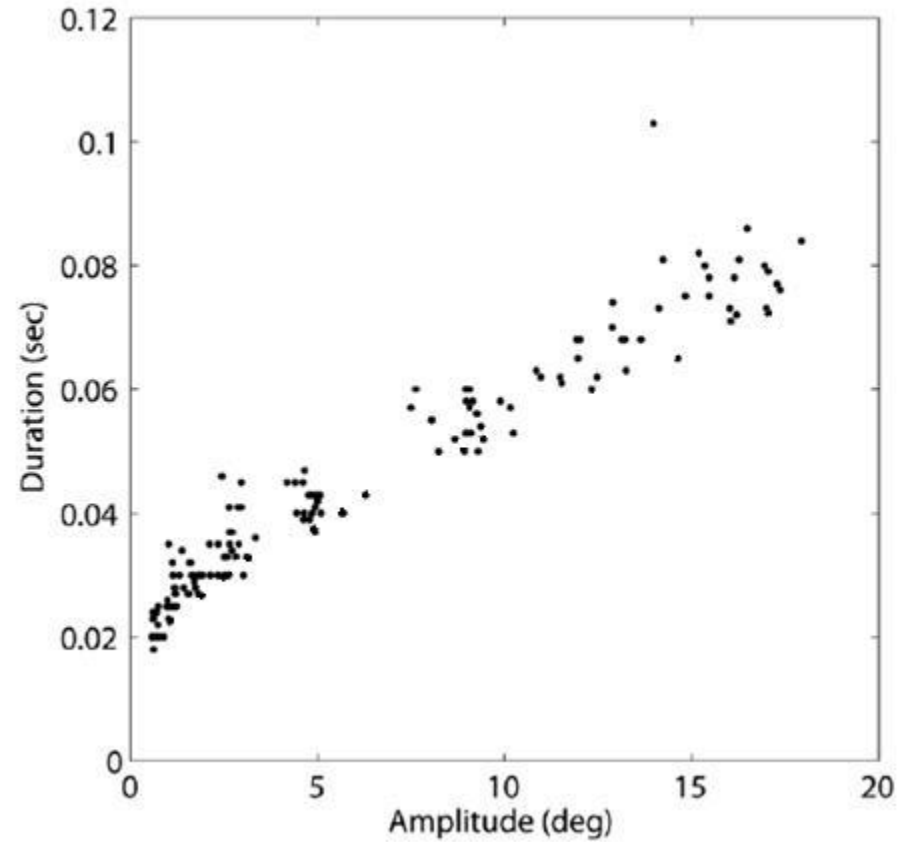
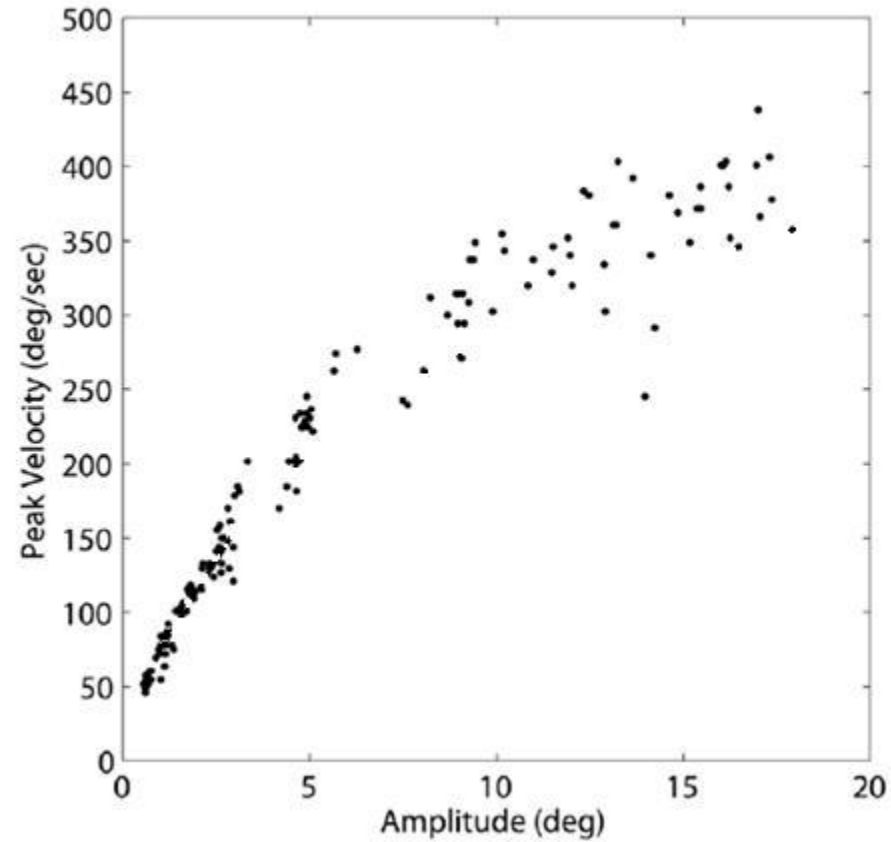
$$\Delta w_{ij}^h = \delta_{ij}^h \int_0^T \text{sign}(r_{\text{vis}}(t)) \left( \int_0^t s_{ij}(\tau) h_h(t - \tau) d\tau \right) dt - n \delta_{ij}^h \alpha_h (w_{ij}^h)^{(n-1)}$$

**Adaptive Parameters:**  $w^e, w^h$

**Free Parameters:**  $n, \alpha_e, \alpha_h, \sigma$

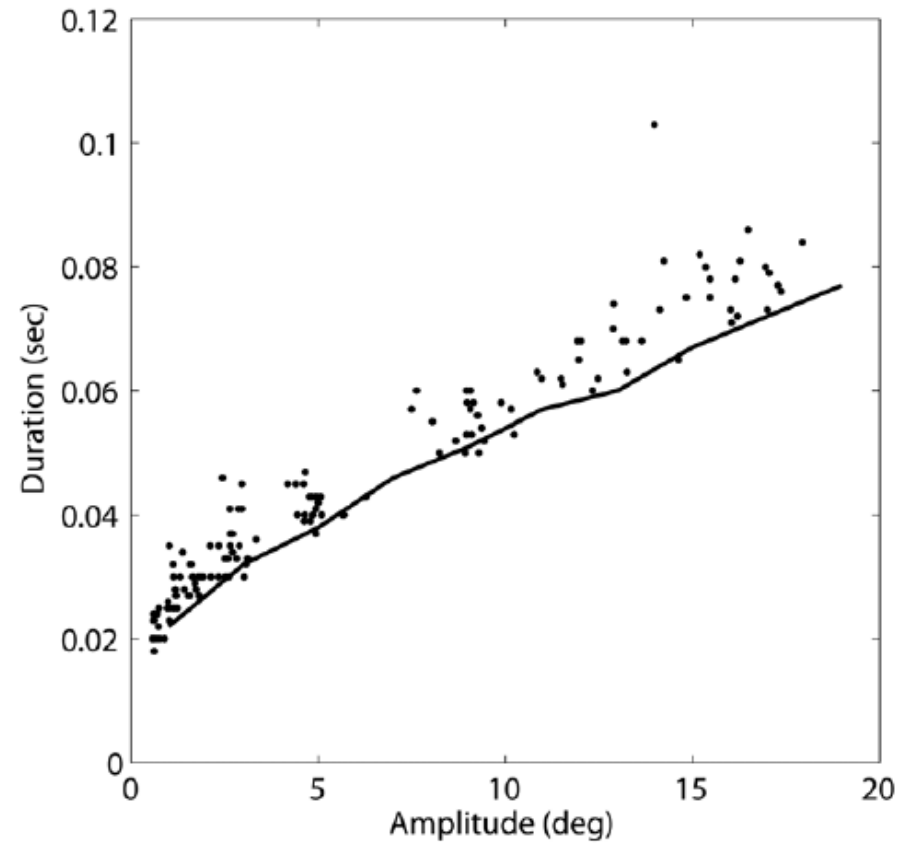
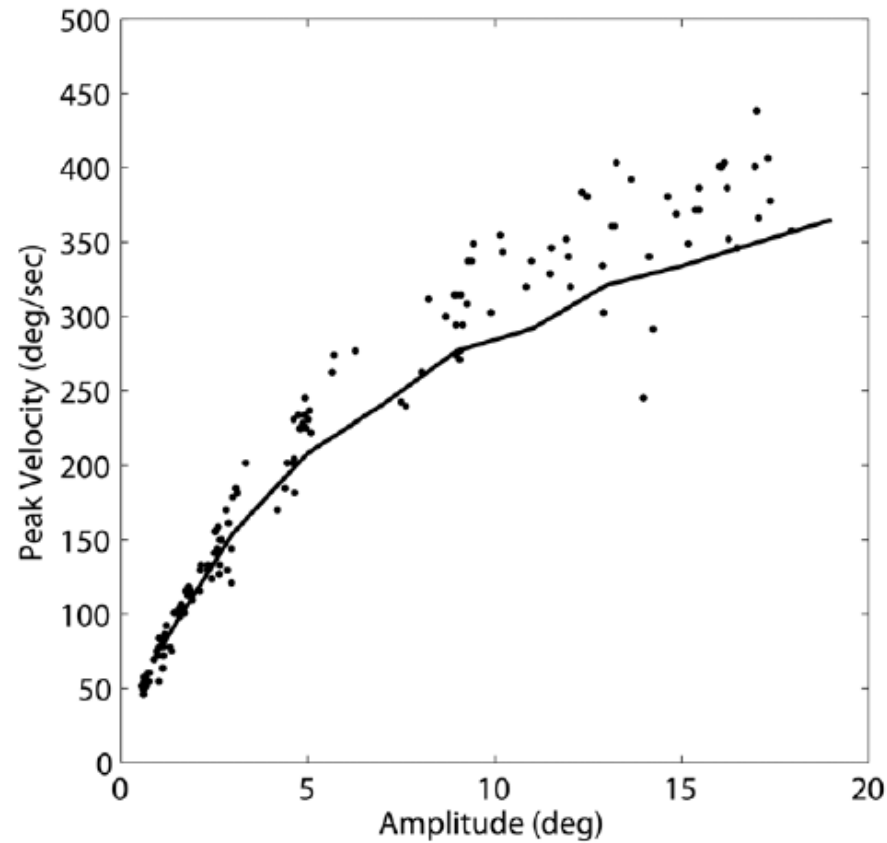


# Results: Head-Restrained Saccades



..... Experimental Data (Harwood et al., 1999)

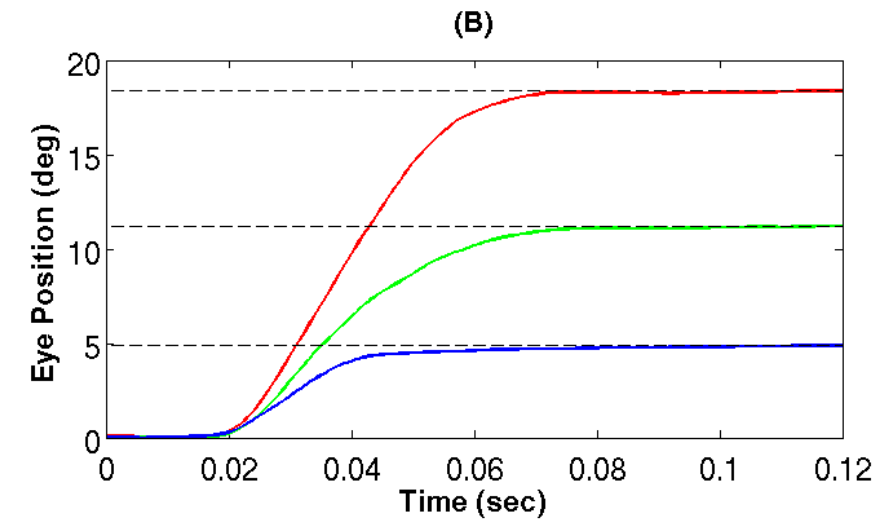
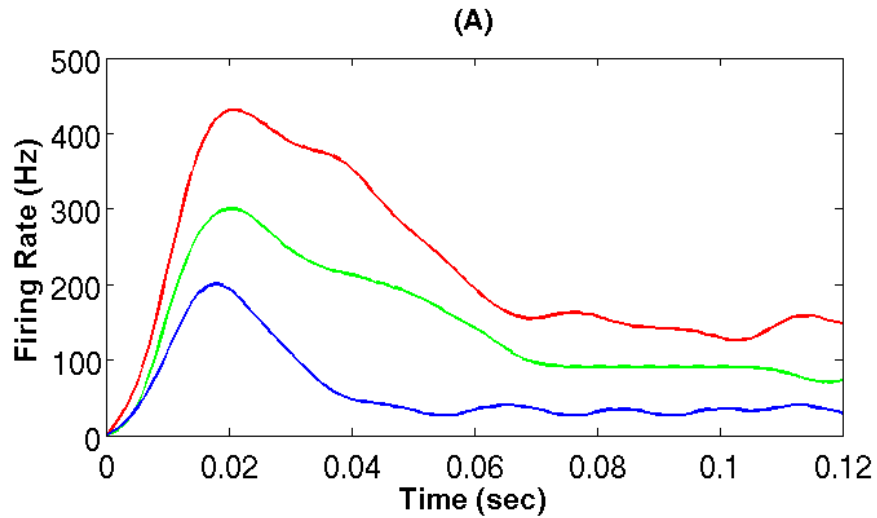
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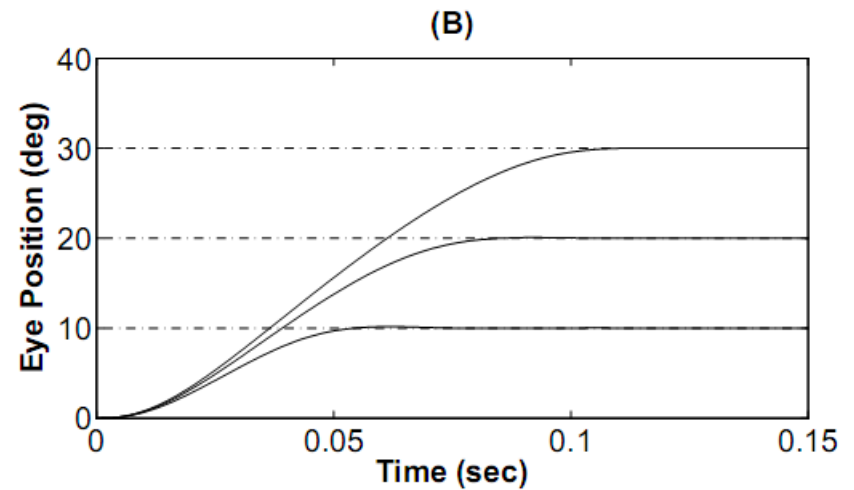
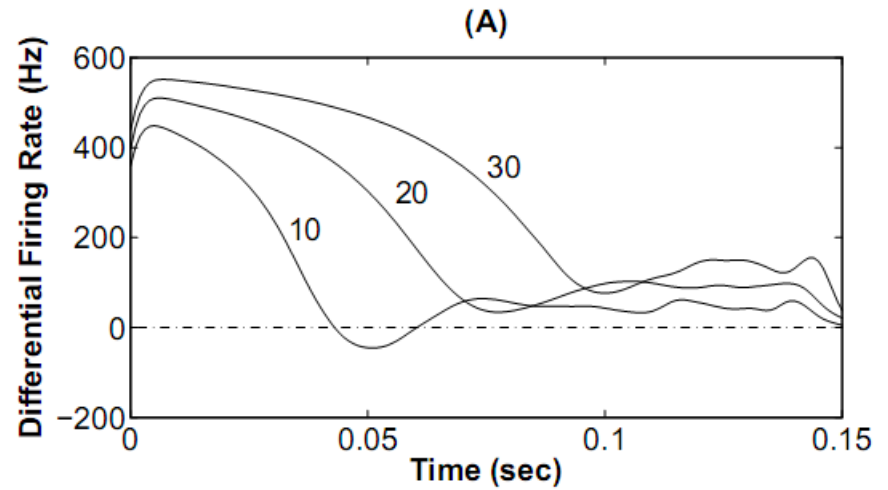
..... Experimental Data (Harwood et al., 1999)

— Model Result ( $\sigma = 0.002$ ,  $\alpha_e = 0.016$ ,  $n=4$ )

# Results: Head-Restrained Saccades

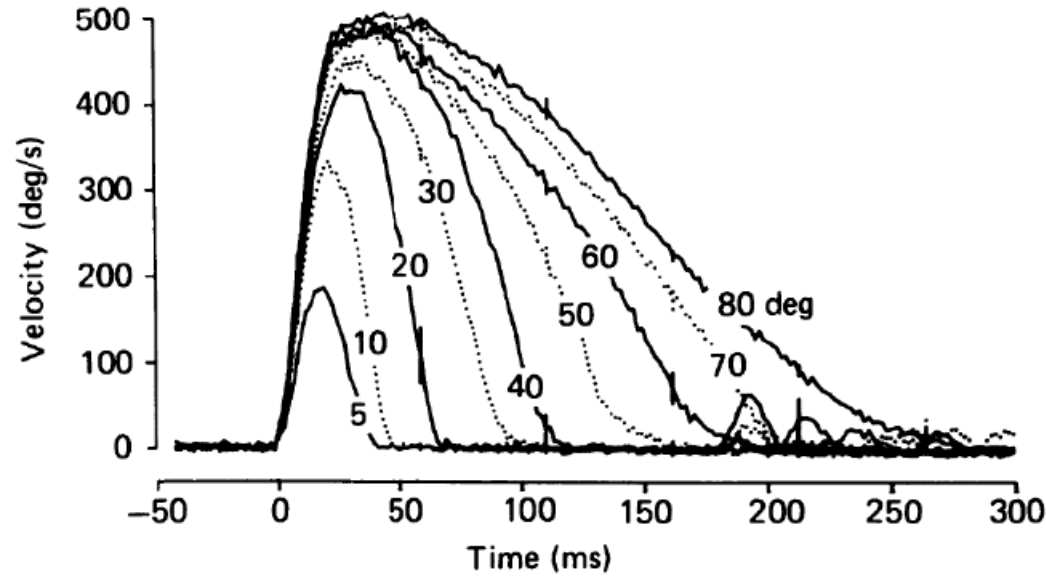


**Experimental Results**  
**(Sylvestre & Cullen, 1999)**



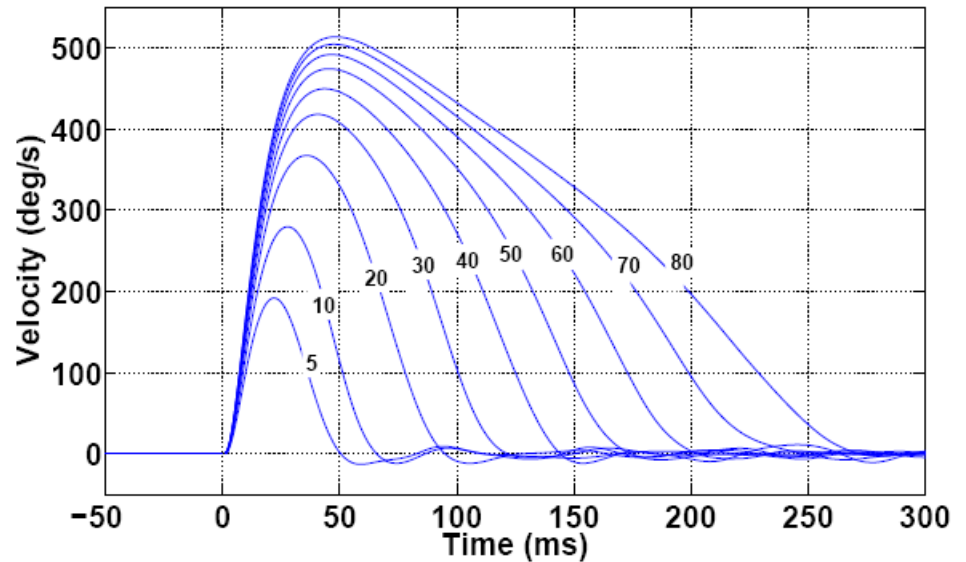
**Model Simulation Results**

# Results: Head-Restrained Saccades

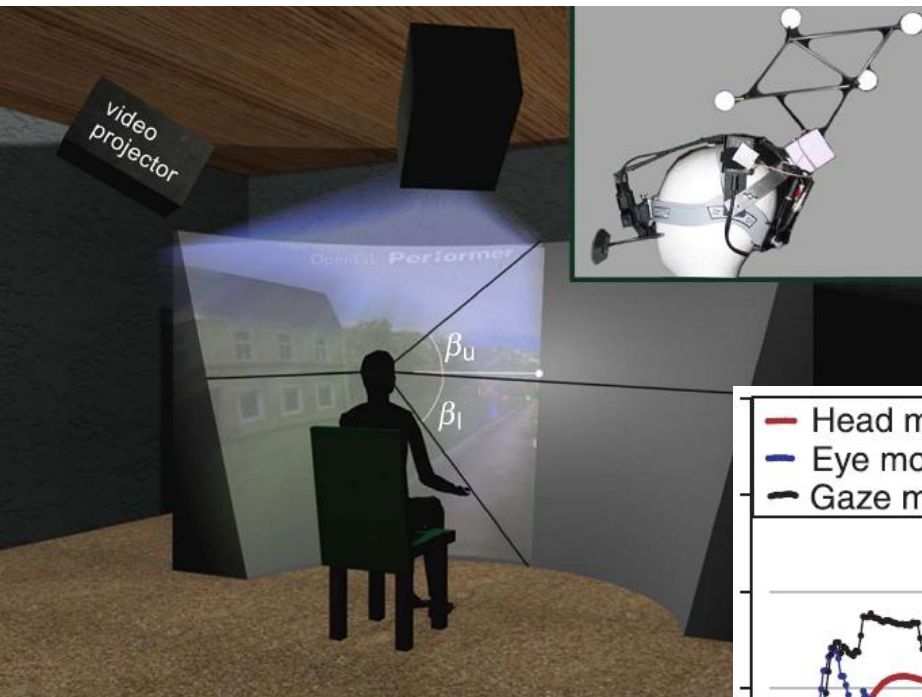


➔ **Experimental Results  
(Collewijn et al., 1988)**

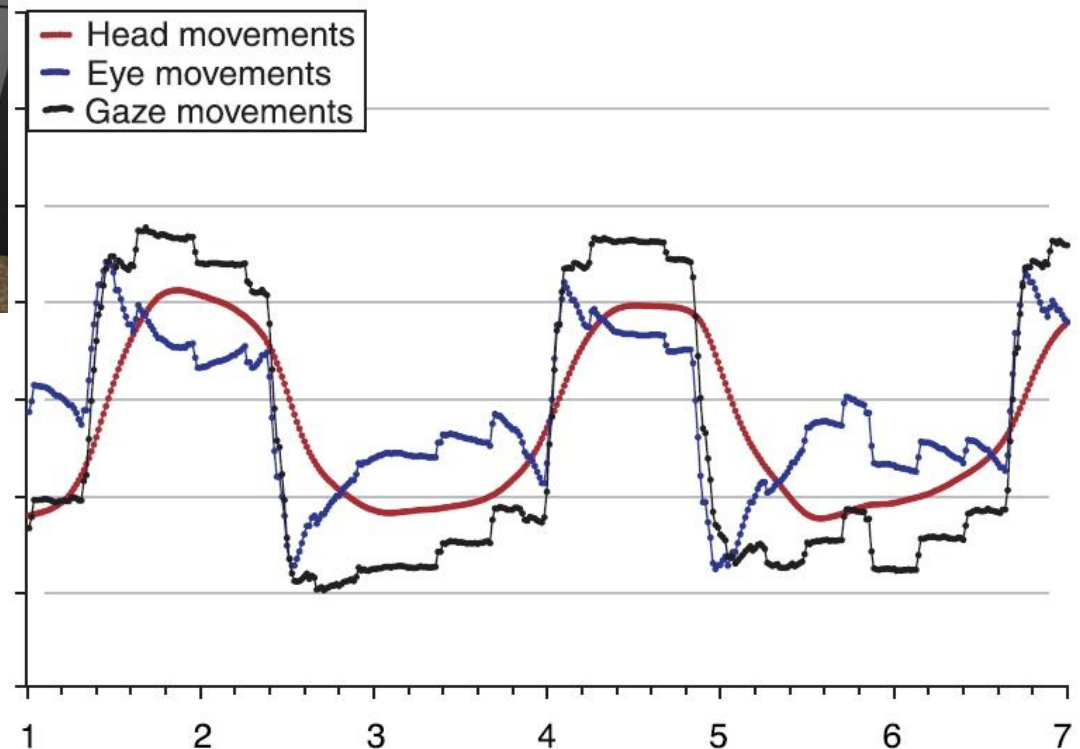
**Model Simulation Results**



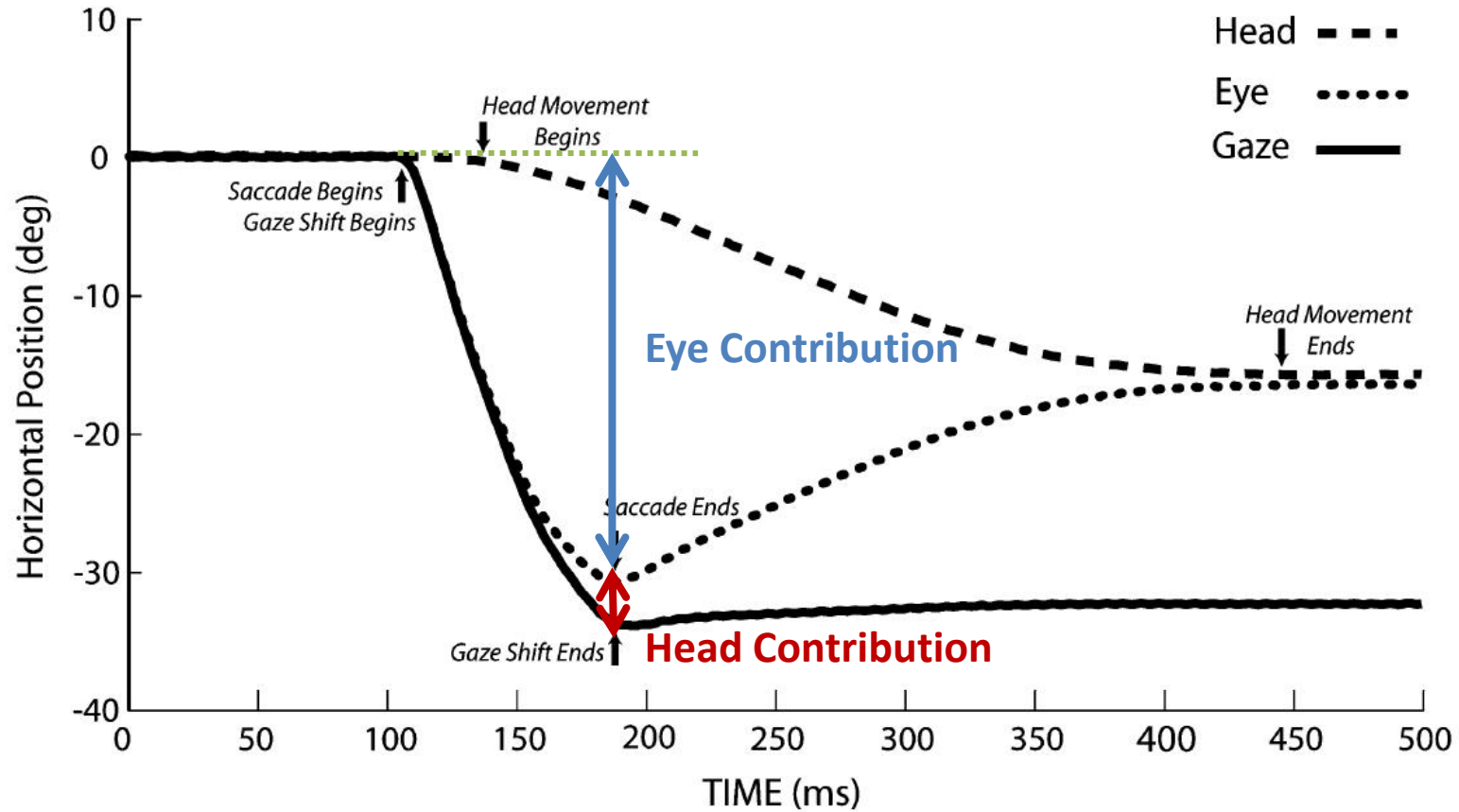
# Results: Head-Free Saccades



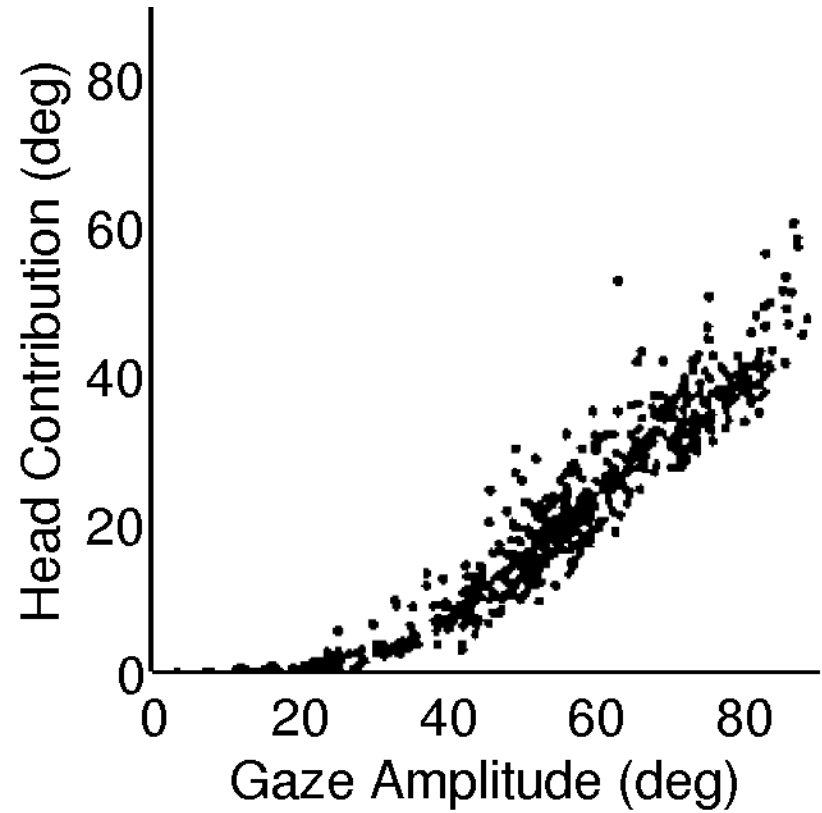
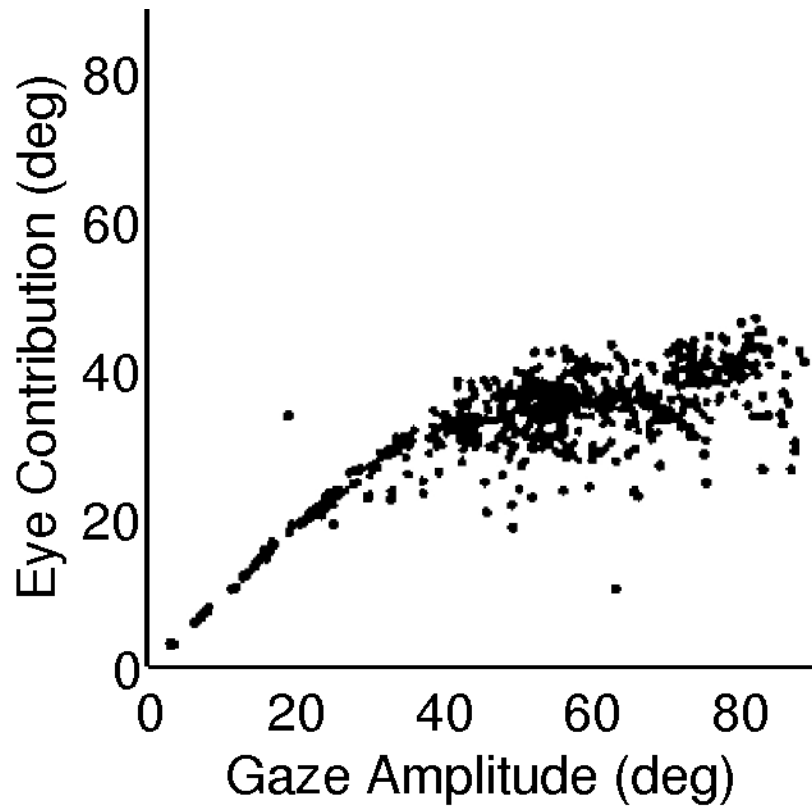
**Head-free gaze shift of human subjects during a visual search paradigm (Hardiess et al, 2008)**



# Results: Head-Free Saccades

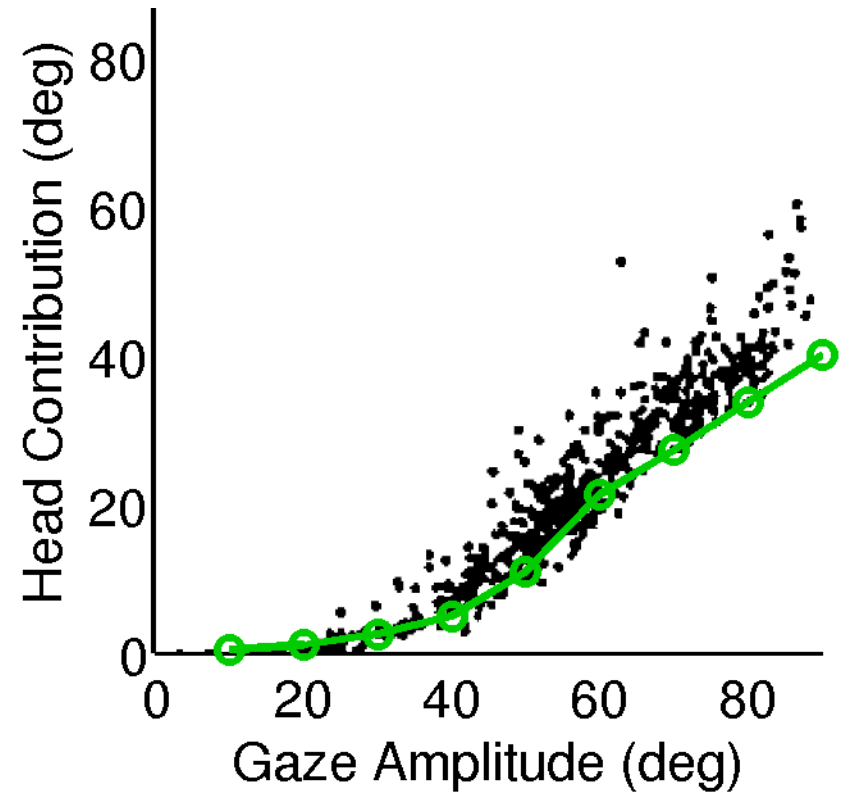
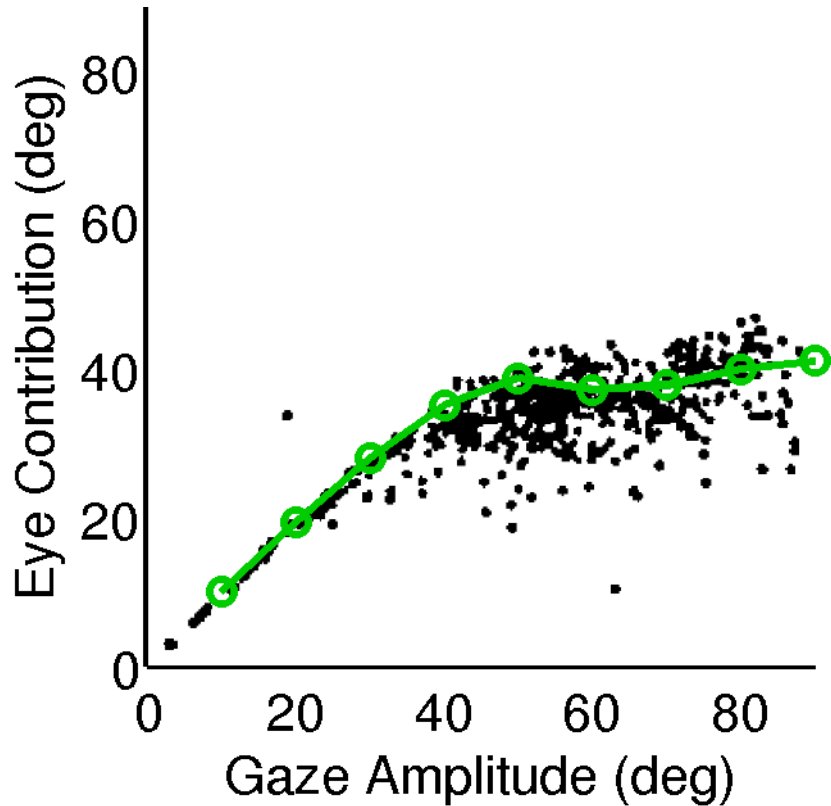


# Results: Head-Free Saccades



..... Experimental Data (Freedman and Sparks; 1997)

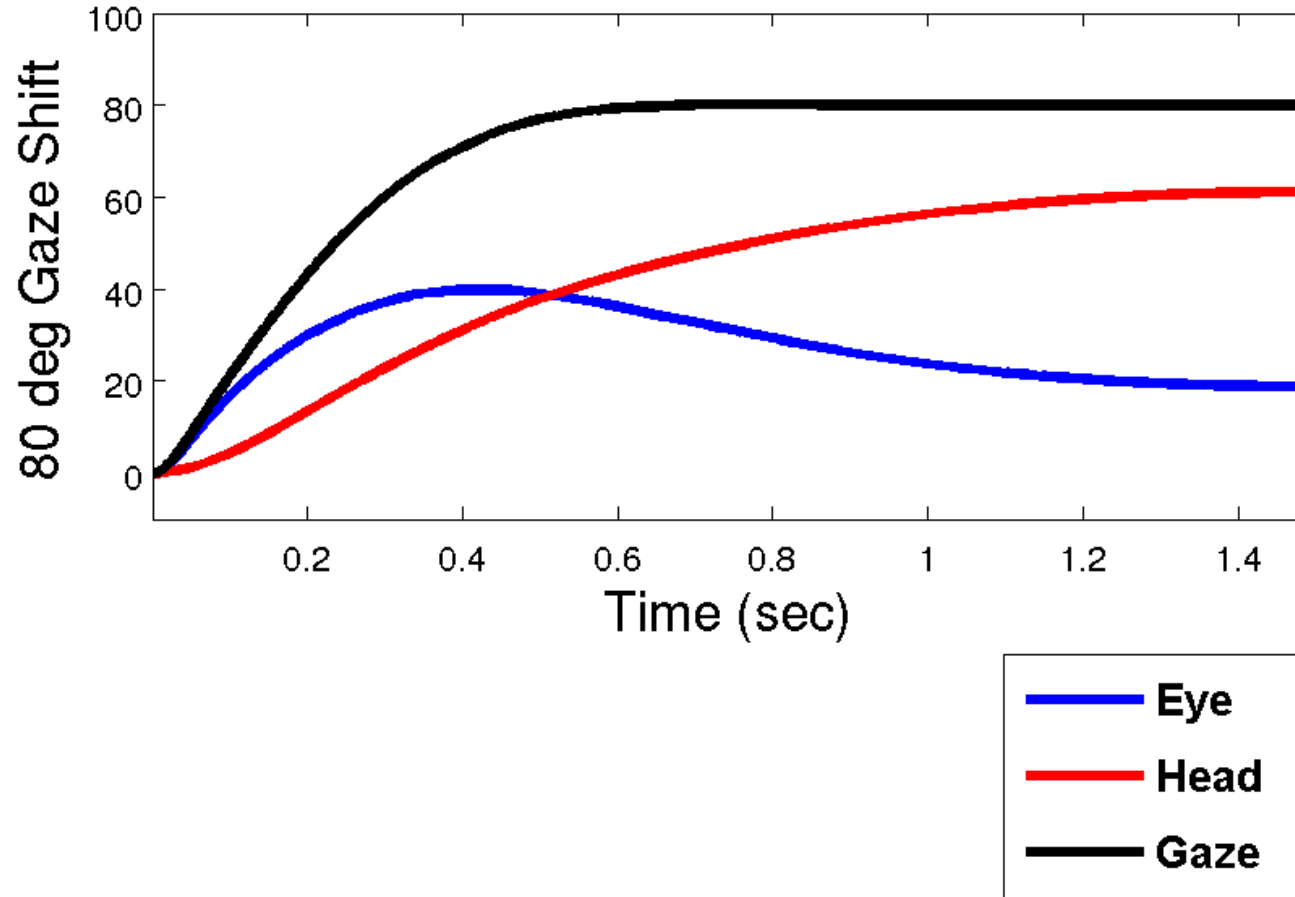
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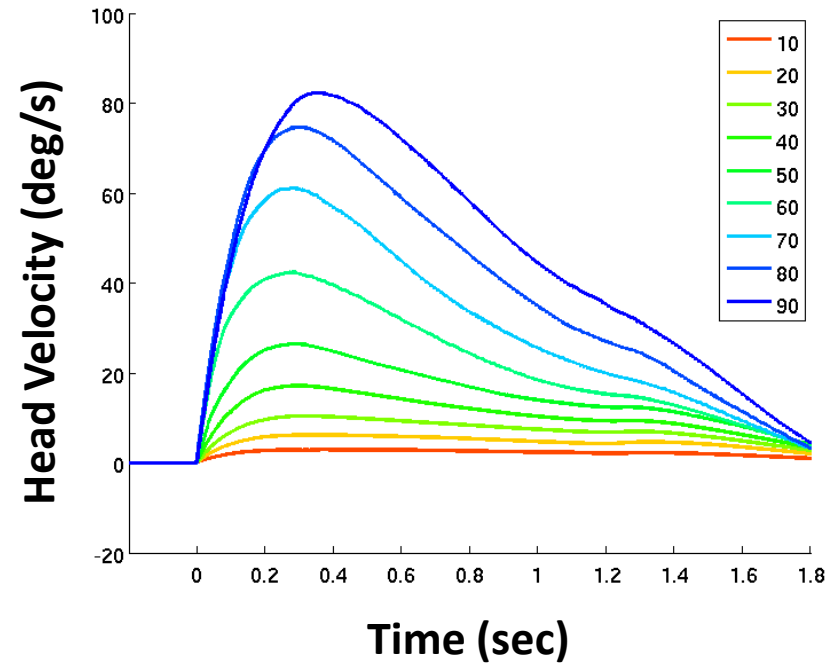
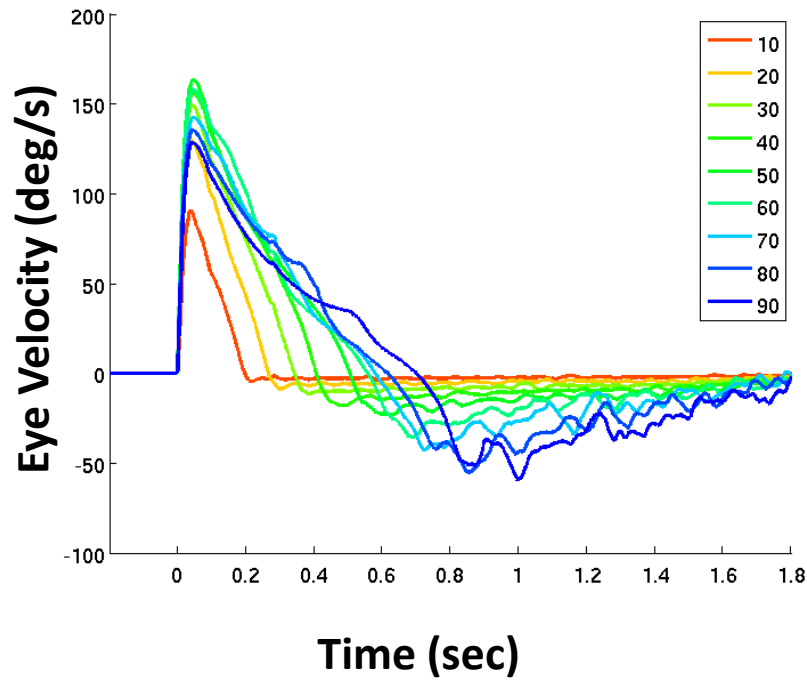
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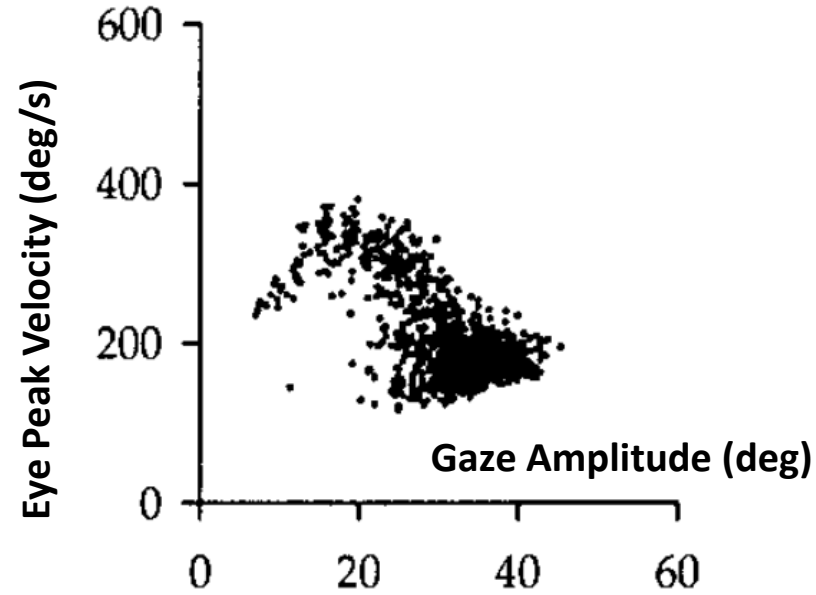


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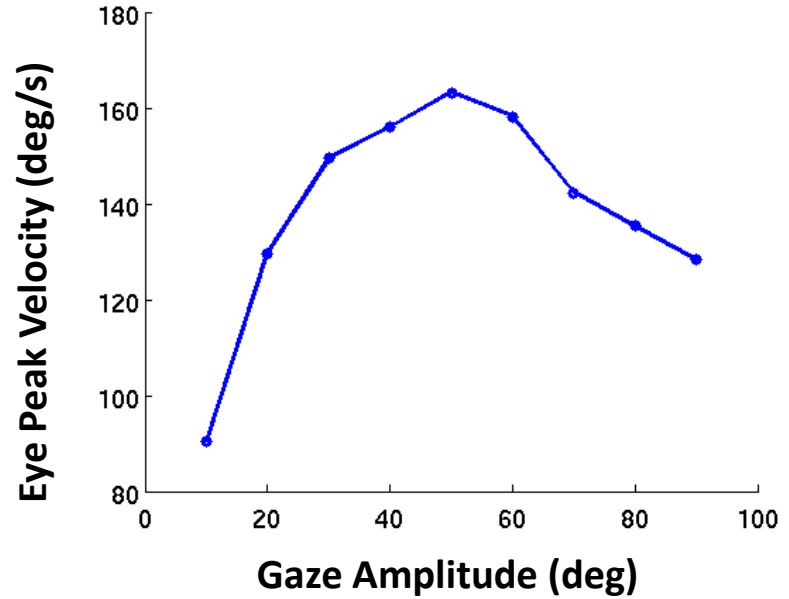


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# Results: Head-Free Saccades



Experimental Data  
(Freedman and Sparks; 1997)



Model Result

# Conclusion

- ➡ Simple open-loop neural controller capable of reproducing biologically realistic eye and head movements.
- ➡ The model is based on an adaptation mechanism that on one hand is local and biologically plausible and on the other hand minimizes a cost function, therefore it creates a bridge between **optimality principles** and **neural architectures**.

## Future Directions:

- ❖ Different initial eye positions
- ❖ Forward model learning
- ❖ Other ballistic movements